DIFERENTIAL EQUATIONS, CLASS EXERCISE 4

(1) Use Euler's method with stepsize $h = 0.2$ to approximate the solution of the initial value problem

$$y' = y + e^{-t}, \quad y(0) = 0$$

on the interval $0 \leq t \leq 1$. Organize your calculations in a table.

Next solve the initial value problem and add a column in your table with the exact values of y at the time points used in the Euler's method. How good is the numerical approximation?

(2) Find a solution for the initial value problem

$$2y'' - 3y' + y = 0, \quad y(0) = 2, \; y'(0) = \frac{1}{2}.$$

Sketch the graph and then determine the maximum value of the solution.

(3) Verify that $y_1(t) = e^t$ and $y_2(t) = te^t$ are solutions of the differential equation

$$y'' - 2y' + y = 0.$$

Do they consitute a fundamental set of solutions? Use Wronskian to justify your answer.

(4) Prove that the three functions $f_1(x) = 1$, $f_2(x) = \sin^2 x$ and $f_3(x) = \cos^2 x$ are dependent by computing their Wronskian.