

Diff Eq - Clex 5 - Solutions

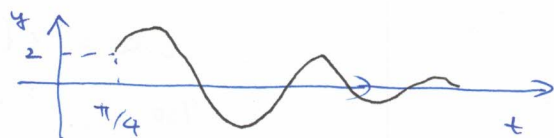
① $r^2 + 2r + 2 = 0$: $r_{1,2} = -1 \pm i$; $y(t) = c_1 e^{-t} \cos t + c_2 e^{-t} \sin t$

$2 = y(\pi/4) = c_1 e^{-\pi/4} \frac{1}{\sqrt{2}} + c_2 e^{-\pi/4} \frac{1}{\sqrt{2}}$

$2 = y'(\pi/4) = c_1 \left(-e^{-\pi/4} \frac{1}{\sqrt{2}} - e^{-\pi/4} \frac{1}{\sqrt{2}} \right) + c_2 \left(-e^{-\pi/4} \frac{1}{\sqrt{2}} + e^{-\pi/4} \frac{1}{\sqrt{2}} \right)$

$c_1 = -\sqrt{2} e^{\pi/4}$, $c_2 = 3\sqrt{2} e^{\pi/4}$

$y(t) = -\sqrt{2} e^{\pi/4 - t} \cos t + 3\sqrt{2} e^{\pi/4 - t} \sin t$



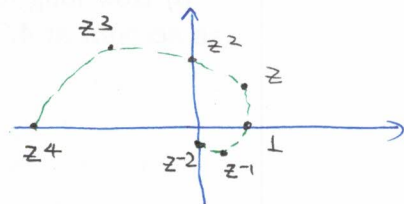
② $x_{1,2} = \frac{-5 \pm \sqrt{25 - 4(4)(2)}}{4} = -\frac{5}{4} \pm \frac{\sqrt{7}}{4} i$, $|x_{1,2}| = \sqrt{\left(\frac{5}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} = \sqrt{2}$

$\arctan\left(-\frac{\sqrt{7}}{5}\right) = 2.655$, $\arctan\left(\frac{\sqrt{7}}{5}\right) = 3.628$

$x_1 = \sqrt{2} e^{2.655i}$, $x_2 = \sqrt{2} e^{3.628i}$

③ $z = 1+i = \sqrt{2} e^{i\pi/4}$, $z^{-2} = \frac{1}{2} e^{-i\pi/2}$, $z^{-1} = \frac{1}{\sqrt{2}} e^{-i\pi/4}$

$z^0 = 1$, $z^2 = 2 e^{i\pi/2}$, $z^3 = 2\sqrt{2} e^{3\pi/4 i}$, $z^4 = 4 e^{i\pi}$



④ $(\cos \theta + i \sin \theta)^n = (e^{i\theta})^n = e^{in\theta} = \cos n\theta + i \sin n\theta$

⑤ Let $z = |z| e^{i\theta}$. Then $|z^n| = |(|z| e^{i\theta})^n| = | |z|^n e^{in\theta} | = |z|^n$

Note that $|e^{in\theta}| = |\cos n\theta + i \sin n\theta| = \sqrt{\cos^2 n\theta + \sin^2 n\theta} = 1$.

⑥ $4r^2 + 4r + 1 = 0$, $(2r+1)^2 = 0$, $r_{1,2} = -1/2$

$y_1(t) = e^{-t/2}$, $y_2(t) = t e^{-t/2}$

$y(t) = c_1 e^{-t/2} + c_2 t e^{-t/2}$

$-1 = y(0) = c_1$, $4 = -c_1/2 + c_2 \Rightarrow c_2 = 7/2$

$y(t) = -e^{-t/2} + \frac{7}{2} t e^{-t/2}$

$\lim_{t \rightarrow \infty} y(t) = 0$

