DIFERENTIAL EQUATIONS, CLASS EXERCISE 5

(1) Solve the initial value problem

$$y'' + 2y' + 2y = 0, \quad y(\pi/4) = 2,\ y'(\pi/4) = 2.$$

Sketch the graph of the solution.

(2) Determine the solutions of the quadratic equation

$$2x^2 + 5x + 4 = 0.$$

Write the two roots in both Cartesian and polar forms.

(3) For $z = 1 + i$ compute the powers z^n for n from $-2 \ldots 4$. Draw these seven complex numbers on the same Cartesian plane.

(4) Prove DeMoivre's identity

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta.$$

(5) Prove that for a complex number z, $|z^n| = |z|^n$.

(6) Solve the initial value problem

$$4y'' + 4y' + y = 0, \ \ y(0) = -1, y'(0) = 4$$

Determine the behaviour of the solution as $t \to \infty$ and sketch the graph.