Diff Eq - Clex6 - Solutions

(1) $y_2(t) = v(t)\, y_1(t)$; $u(t) = v'(t)$

$y_1 u' + (2y_1' + p y_1)u = 0$; $u' + (2\, y_1'/y_1 + p)u = 0$

$u(t) = e^{-\int (2y_1'/y_1 + p)\,dt} = e^{-\int (2\cdot(-1/t^2)/(1/t) + 3/t)\,dt} = e^{-\ln t} = t^{-1}$

$\Rightarrow v(t) = \ln|t| \Rightarrow y_2(t) = \ln|t|/t.$

(2) $Y(t) = At + B$, $Y' = A$, $Y'' = 0 \Rightarrow A - 2At - 2B = 2t \Rightarrow A = -1,\ B = -1/2$

$Y(t) = -t - 1/2$. Solve the homogeneous equation. $r^2 + r - 2 = 0$, $r_1 = 1$, $r_2 = -2$

$y(t) = c_1 e^t + c_2 e^{-2t} - t - 1/2$; $0 = y(0) = c_1 + c_2 - 1/2$, $1 = y'(0) = c_1 - 2c_2 - 1$

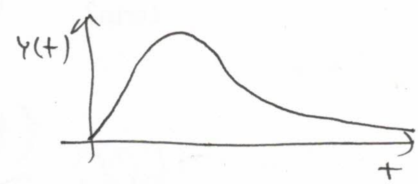
$\Rightarrow c_2 = -1/2,\ c_1 = 1 \Rightarrow y(t) = -\frac{1}{2} e^{-2t} + e^t - t - 1/2$

(3) $r^2 + 2r + 1 = 0 \Rightarrow r_1 = r_2 = -1$; $y_1(t) = e^{-t}$, $y_2(t) = te^{-t}$

$$W(y_1, y_2) = \begin{vmatrix} e^{-t} & te^{-t} \\ -e^{-t} & e^{-t} - te^{-t} \end{vmatrix} = e^{-2t}$$

$$u_1 = -\int \frac{te^{-t}\cdot 3e^{-t}}{e^{-2t}}\,dt = -\frac{3t^2}{2} ; \quad u_2 = \int \frac{e^{-t}\, 3e^{-t}}{e^{-2t}}\,dt = 3t$$

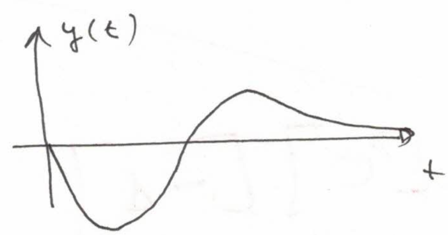
$$Y(t) = -\frac{3t^2}{2} e^{-t} + 3t\cdot t\, e^{-t} = \frac{3}{2} t^2 e^{-t}$$



$$y(t) = c_1 e^{-t} + c_2 te^{-t} + \frac{3}{2} t^2 e^{-t}$$

$y(0) = c_1 = 0$, $y'(0) = -c_1 + c_2 = -4 \Rightarrow c_2 = -4$

$$y(t) = \left(\frac{3}{2} t^2 - 4t\right) e^{-t}$$



(4) For $y_1(t) = e^t$, $y_1' = e^t$, $y_1'' = e^t$: $(1-t)e^t + te^t - e^t = 0$ ✓

For $y_2(t) = t$, $y_2' = 1$, $y_2'' = 0$: $(1-t)\cdot 0 + t\cdot 1 - t = 0$ ✓

$$W(y_1, y_2) = \begin{vmatrix} e^t & t \\ e^t & 1 \end{vmatrix} = e^t(1-t) \neq 0$$

Put the DE in standard form: $y'' + \frac{t}{1-t} y' - \frac{1}{1-t} y = -2(1-t)e^{-t}$

$$u_1(t) = -\int \frac{y_2(t)\, g(t)}{W(y_1, y_2)}\,dt = -\int \frac{t\cdot 2(1-t)e^{-t}}{e^t(1-t)}\,dt = +te^{-2t} + \frac{1}{2} e^{-2t} + c_1$$

Diff Eq - Clex 6 - Solutions

$$u_2(t) = \int \frac{y_1(t)\, g(t)}{W(y_1, y_2)}\, dt = \int \frac{e^t \cdot 2(1-t)e^{-t}}{e^t(1-t)}\, dt = -2e^{-t} + C_2$$

$$Y(t) = u_1(t)\, y_1(t) + u_2(t)\, y_2(t) = \left[+t e^{-2t} + \frac{1}{2} e^{-2t}\right] e^t - 2e^{-t}\, t$$

$$Y(t) = \left(1/2 - t\right) e^{-t}$$