DIFERENTIAL EQUATIONS, CLASS EXERCISE 6

(1) Use the method of reduction of order to find a second solution of the given differential equation

$$t^2y'' + 3ty' + y = 0, \quad y_1(t) = t^{-1}.$$

(2) Use the method of undetermined coefficients to find the solution of the initial value problem

$$y'' + y' - 2y = 2t$$
, $y(0) = 0$, $y'(0) = 1$.

(3) Use the method of variation of parameters to find a particular solution of the differential equation

$$y'' + 2y' + y = 3e^{-t}.$$

Sketch the graph of this particular solution.

Next, solve the initial value problem y(0) = 0, y'(0) = -4 for the same DE. Sketch the graph of the resulting solution.

(4) Verify that the functions $y_1(t) = e^t$ and $y_2(t) = t$ are solutions of the homogeneous equation corresponding to

$$(1-t)y'' + ty' - y = 2(t-1)^2 e^{-t}.$$

Next find a particular solution of the nonhomogeneous equation above.