DIFERENTIAL EQUATIONS, CLASS EXERCISE 7

(1) A series circuit has a capacitor of $10^{-5}F$, a resistor of 300Ω and an inductor of $0.2H$. The initial charge on the capacitor is $10^{-6}C$ and there is no initial current. Find the charge $Q(t)$ on the capacitor as a function of time.

(2) Consider a vibrating system described by an initial value problem

$$u'' + \frac{1}{4}u' + 2u = 2\cos(\omega t), \quad u(0) = 0, u'(0) = 2.$$

In all three parts below do the computations from first principles without using the canned formulas from the notes (or the book).

i) Descibe the steady state part of the solution of this problem.

ii) Find the amplitude of the steady state solution in terms of ω.

iii) Find the maximum value of A and the frequency ω for which it occurs.

(3) A frictionless mass-spring system is driven by a periodic external force. The differential equation describing the dynamics is

$$u'' + 4u = 3\cos(1.8t)$$

The initial conditions are $u(0) = 0, u'(0) = 0$.

i) Derive the solution of this IVP.

ii) Express the solution as a beat with slowly varying amplitude.

iii) What is the amplitude of the envelope of the beat?

iv) Sketch a picture of the solution.

(4) Consider the mechanical system drawn below. Assuming that the masses m_1 and m_2 experience friction with friction coeffiecients γ_1 and γ_2, respectively, write the equations of motion for the displacements from equilibrium position, u_1 and u_2. Next, by introducing new variables, rewrite the equations of motion as a system of first order differential equations. Finally, write the first order system in matricial form.