

Clax 8 - solutions

$$\textcircled{1} \det(A - \lambda I) = \begin{vmatrix} -1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 2 \\ 0 & 2 & 1-\lambda \end{vmatrix} = (-1-\lambda)[(1-\lambda)^2 - 4] = -(\lambda+1)^2(\lambda-3)$$

$$\lambda_1 = \lambda_2 = -1, \lambda_3 = 3$$

$$\lambda_1 = \lambda_2 = -1 \quad \left( \begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad X = \begin{pmatrix} t \\ -s \\ 0 \end{pmatrix} = t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, X_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$\lambda_3 = 3 \quad \left( \begin{array}{ccc|c} -4 & 0 & 0 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 2 & -2 & 0 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad X = \begin{pmatrix} 0 \\ u \\ u \end{pmatrix}, X_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\textcircled{2} \det(B - \lambda I) = \begin{vmatrix} 1-\lambda & 2 \\ -1 & -1-\lambda \end{vmatrix} = \lambda^2 + 1 = 0 \quad \lambda_1 = i, \lambda_2 = -i$$

$$\lambda_1 = i \quad \left( \begin{array}{cc|c} 1-i & 2 & 0 \\ -1 & -1-i & 0 \end{array} \right) \rightarrow \left( \begin{array}{cc|c} 1 & 1+i & 0 \\ 0 & 0 & 0 \end{array} \right) \quad X = \begin{pmatrix} -(1+i)s \\ s \end{pmatrix} \rightarrow X_1 = \begin{pmatrix} -1-i \\ 1 \end{pmatrix}$$

The second pair is complex conjugate

$$\lambda_2 = -i \quad X_2 = \begin{pmatrix} -1+i \\ 1 \end{pmatrix}$$

$$\textcircled{3} \begin{vmatrix} -1.5-\lambda & 0.5 \\ 1 & -1-\lambda \end{vmatrix} = \lambda^2 - 2.5\lambda + 1, \quad \lambda_1 = -0.5, \lambda_2 = -2$$

$$\lambda_1 = -0.5 \quad \left( \begin{array}{cc|c} -1 & 0.5 & 0 \\ 1 & -0.5 & 0 \end{array} \right) \rightarrow \left( \begin{array}{cc|c} 1 & -0.5 & 0 \\ 0 & 0 & 0 \end{array} \right) \quad X = \begin{pmatrix} s/2 \\ s \end{pmatrix} \quad X_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

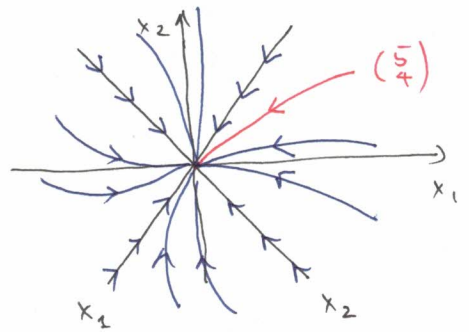
$$\lambda_2 = -2 \quad \left( \begin{array}{cc|c} 0.5 & 0.5 & 0 \\ 1 & 1 & 0 \end{array} \right) \rightarrow \left( \begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right) \quad X = \begin{pmatrix} -t \\ t \end{pmatrix} \quad X_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{X}(t) = c_1 e^{-0.5t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{X}(0) = \begin{pmatrix} 5 \\ 4 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \Rightarrow \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\vec{X}(t) = 3e^{-0.5t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + (-2)e^{-2t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$



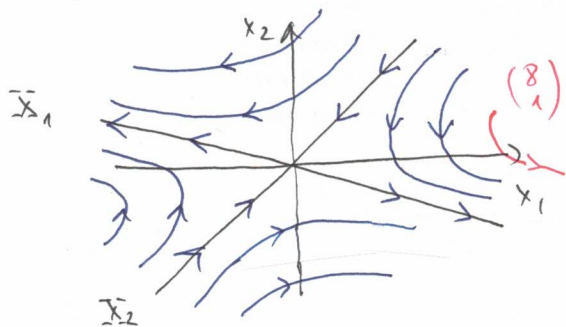
# Clex 8 - Solutions

$$\textcircled{4} \quad \begin{vmatrix} 4-\lambda & -5 \\ -2 & 1-\lambda \end{vmatrix} = \lambda^2 - 5\lambda - 6 \quad \lambda_1 = 6, \lambda_2 = -1$$

$$\lambda_1 = 6 \quad \begin{pmatrix} -2 & -5 & | & 0 \\ -2 & -5 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 5/2 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \quad X = \begin{pmatrix} -5/2s \\ s \end{pmatrix} \rightarrow X_1 = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

$$\lambda_2 = -1 \quad \begin{pmatrix} 5 & -5 & | & 0 \\ -2 & 2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \quad X = \begin{pmatrix} t \\ t \end{pmatrix} \rightarrow X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\vec{x}(t) = c_1 e^{6t} \begin{pmatrix} -5 \\ 2 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



$$\vec{x}(0) = \begin{pmatrix} 8 \\ 1 \end{pmatrix} = c_1 \begin{pmatrix} -5 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -5 & 1 \\ 2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 8 \\ 1 \end{pmatrix} = \frac{1}{(-7)} \begin{pmatrix} 1 & -1 \\ -2 & -5 \end{pmatrix} \begin{pmatrix} 8 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\vec{x}(t) = -e^{6t} \begin{pmatrix} -5 \\ 2 \end{pmatrix} + 3e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\textcircled{5} \quad \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = (1-\lambda)[(1-\lambda)(-\lambda)-1] + (1)[-(1-\lambda)] = -(\lambda-2)(\lambda+1)(\lambda-1)$$

$$\lambda_1 = -1 \quad \lambda_2 = 1 \quad \lambda_3 = 2$$

$$\lambda_1 = -1 \quad \begin{pmatrix} 2 & 0 & 1 & | & 0 \\ 0 & 2 & 1 & | & 0 \\ 1 & 1 & 1 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1/2 & | & 0 \\ 0 & 1 & 1/2 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \quad X_1 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

$$\lambda_2 = 1 \quad \begin{pmatrix} 0 & 0 & 1 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 1 & 1 & -1 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \quad X_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\lambda_3 = 2 \quad \begin{pmatrix} -1 & 0 & 1 & | & 0 \\ 0 & -1 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \quad X_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{x}(t) = c_1 e^{-t} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} + c_2 e^t \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + c_3 e^{2t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$