DIFERENTIAL EQUATIONS, CLASS EXERCISE 8

(1) Compute the eigenvalues and the eigenvectors of the matrix A

$$A = \left(\begin{array}{rrrr} -1 & 0 & 0\\ 0 & 1 & 2\\ 0 & 2 & 1 \end{array}\right).$$

(2) Compute the eigenvalues and the eigenvectors of the matrix B

$$B = \left(\begin{array}{cc} 1 & 2\\ -1 & -1 \end{array}\right).$$

(3) Solve the initial value problem $\mathbf{x}'(t) = A\mathbf{x}(t)$,

$$A = \begin{pmatrix} -1.5 & 0.5 \\ 1 & -1 \end{pmatrix}, \qquad \mathbf{x}(0) = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

Draw the phase portrait of this linear system of DE's emphasizing the particular trajectory selected by the initial conditions.

(4) a) Solve the initial value problem $\mathbf{x}'(t) = A\mathbf{x}(t)$,

$$A = \begin{pmatrix} 4 & -5 \\ -2 & 1 \end{pmatrix}, \qquad \mathbf{x}(0) = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$$

Draw the phase portrait of this linear system of DE's emphasizing the particular trajectory selected by the initial conditions.

(5) Determine the general solution of $\mathbf{x}'(t) = A\mathbf{x}(t)$,

$$A = \left(\begin{array}{rrr} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{array} \right).$$