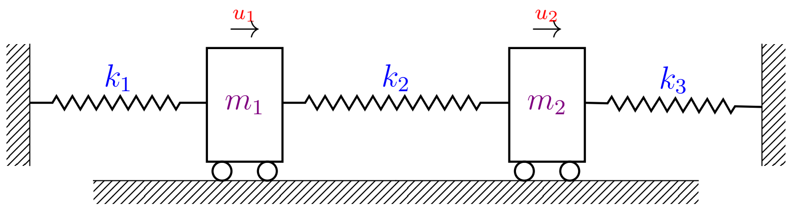
DIFERENTIAL EQUATIONS, CLASS EXERCISE 9

(1) Solve the initial value problem $\mathbf{x}'(t) = A\mathbf{x}(t)$,

$$A = \begin{pmatrix} -2 & -2.5 \\ 10 & -2 \end{pmatrix}, \qquad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 3 \end{pmatrix}.$$

Draw the phase portrait of this linear system of DE's emphasizing the particular trajectory selected by the initial conditions.

(2) Consider the two-mass, three-spring system drawn below.



Let $m_1 = 1, m_2 = 4/3, k_1 = 1, k_2 = 3$, and $k_3 = 4/3$.
a) Convert the dynamical equatins of this system to four first order DE's and then write them in the form $u' = Au$.
b) Using software, find the eigenvalues and the eigenvectors of A.
c) Write down the general solution of the system.
d) Describe the four fundamental modes of vibration as four-vectors of functions and also in English.
e) For each fundamental mode draw graphs of the displacements u_1 and u_2 versus t on the same graph.

(3) The coefficient matrix of the following system of differential equations depends on a parameter α.
a) Determine the eigenvalues in terms of α.
b) Find the critical values of α where the qualitative nature of the phase portrait for the system changes.
c) Draw qualitative phase portraits for this system for α at the critical points and also for values of α taken in the intervals between the critical points.

$$\mathbf{x}' = \begin{pmatrix} 3 & \alpha \\ -6 & -4 \end{pmatrix}\mathbf{x}.$$

Hint: Do not miss the critical value where one of the eigenvalues becomes zero.

(4) Consider the system of linear DE's

$$\mathbf{x}' = A\mathbf{x}, \quad A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}.$$

i) Compute the fundamental matrix $\Phi(t) = e^{At}$.
ii) Solve this system with the initial conditions

$$\mathbf{x}(0) = \begin{pmatrix} -1 \\ 3 \end{pmatrix}.$$