Diff eq - §33 - Competing species

Two similar species complete for a limited food supply (They do not prey on each other.

In the absence of competition the two species populations follow logistic growth each:

$$\frac{dx}{dt} = x(\varepsilon_1 - \sigma_1 x), \qquad \frac{dy}{dt} = y(\varepsilon_2 - \sigma_2 y)$$

In the presence of competition the dynamics becomes

$$\frac{dx}{dt} = x(\varepsilon_1 - \sigma_1 x - \alpha_1 y), \qquad \frac{dy}{dt} = y(\varepsilon_2 - \sigma_2 y - \alpha_2 x)$$

$\alpha_{1,2}$ are measures of the degree in each one of the species inteferes with the other.

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ex:

$$\frac{dx}{dt} = x(1-x-y), \qquad \frac{dy}{dt} = y(0.75-y-0.5x)$$

To find the critical points we solve the algebraic system

$$x(1-x-y)=0 \qquad y(0.75-y-0.5x)=0$$

The four critical points are

$$\underbrace{\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}0\\0.75\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}}_{\text{one of the species is absent}}, \underbrace{\begin{pmatrix}0.5\\0.5\end{pmatrix}}_{\uparrow\text{ both species are present.}}$$

To investigate the behaviour of the system we need the Jacobian:

$$J = \begin{pmatrix} 1-2x-y & -x \\ -0.5y & 0.75-2y-0.5x \end{pmatrix}$$

At $\begin{pmatrix}0\\0\end{pmatrix}$: $\begin{pmatrix}x\\y\end{pmatrix}' = \begin{pmatrix}1 & 0\\0 & 0.75\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$, $\lambda_1 = 1$, $\lambda_2 = 0.75$ unstable

At $\begin{pmatrix}0\\0.75\end{pmatrix}$: $\begin{pmatrix}u\\v\end{pmatrix}' = \begin{pmatrix}0.25 & 0\\-0.375 & -0.75\end{pmatrix}\begin{pmatrix}u\\v\end{pmatrix}$, $\lambda_1 = 0.25$, $\lambda_2 = -0.75$ unstable

At $\begin{pmatrix}1\\0\end{pmatrix}$: $\begin{pmatrix}u\\v\end{pmatrix}' = \begin{pmatrix}-1 & -1\\0 & 0.25\end{pmatrix}\begin{pmatrix}u\\v\end{pmatrix}$, $\lambda_1 = -1$, $\lambda_2 = 0.25$ unstable

At $\begin{pmatrix}0.5\\0.5\end{pmatrix}$: $\begin{pmatrix}u\\v\end{pmatrix}' = \begin{pmatrix}-0.5 & -0.5\\-0.25 & -0.5\end{pmatrix}\begin{pmatrix}u\\v\end{pmatrix}$, $\lambda_1 \approx -0.146$, $\lambda_2 \approx -0.854$
Asymptotically stable.

See appendix.

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Ex: $\frac{dx}{dt} = x(1-x-y)$, $\frac{dy}{dt} = y(0.5 - 0.25y - 0.75x)$

The critical points are: $\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}0.5\\0.5\end{pmatrix}$.

The Jacobian is

$$J = \begin{pmatrix} 1-2x-y & -x \\ -0.75y & 0.5-0.5y-0.75x \end{pmatrix}$$

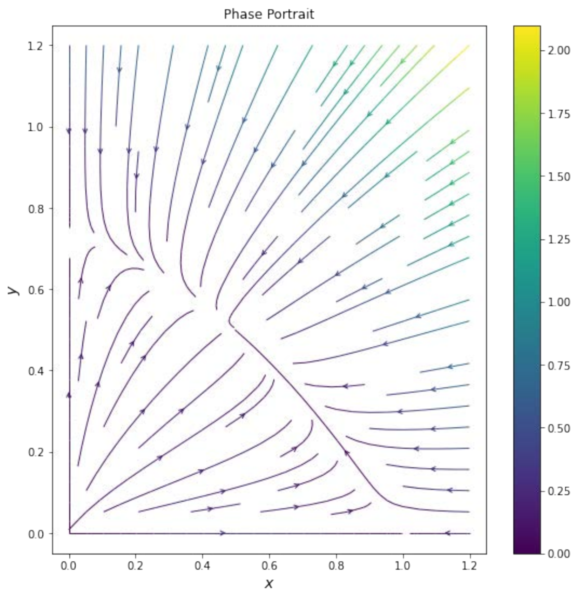
At $\begin{pmatrix}0\\0\end{pmatrix}$: $J = \begin{pmatrix}1 & 0\\ 0 & 0.5\end{pmatrix}$, $\lambda_1 = 1$, $\lambda_2 = 0.5$ Unstable.

At $\begin{pmatrix}1\\0\end{pmatrix}$: $J = \begin{pmatrix}-1 & -1\\ 0 & -0.25\end{pmatrix}$, $\lambda_1 = -1$, $\lambda = -0.25$ Asymptotically stable.

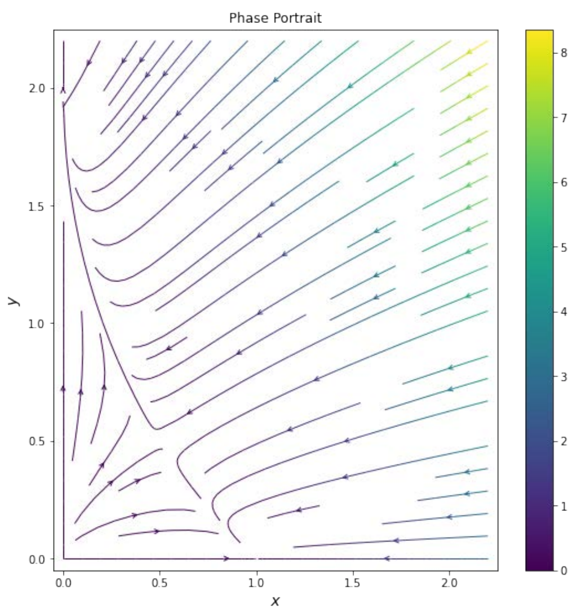
At $\begin{pmatrix}0\\2\end{pmatrix}$: $J = \begin{pmatrix}-1 & 0\\ -1.5 & -0.5\end{pmatrix}$, $\lambda_1 = -1$, $\lambda_2 = -0.5$ Asymptotically stable.

At $\begin{pmatrix}0.5\\0.5\end{pmatrix}$: $J = \begin{pmatrix}-0.5 & -0.5\\ -0.375 & -0.125\end{pmatrix}$ $\lambda_1 \approx 0.1594$, $\lambda_2 \approx -0.7844$ Unstable.

The phase diagram (see Appendix) has a separatrix which divides the 1st quadrant into two basins of attraction: That of $\begin{pmatrix}1\\0\end{pmatrix}$ and that of $\begin{pmatrix}0\\2\end{pmatrix}$. So in this example there is no stable coexistence state.



Phase portrait for the system on page 0.



Phase portrait for the system on page 1.