The general first order equation is $M(x,y) + N(x,y)\frac{dy}{dx} = 0$. (No longer linear)! If it so happens that $M(x,y) = M(x)$ and $N(x,y) = N(y)$ then

$$M(x) + N(y)\frac{dy}{dx} = 0$$

Such an equation is separable: $M(x)dx + N(y)dy = 0$.

Ex: Solve $\frac{dy}{dx} = \frac{x^2}{1-y^2}$, $y(0) = 1$

$\int (1-y^2)\,dy = \int x^2 dx$, $y - y^3/3 = x^3/3 + c$, $3y - y^3 = x^3 + k$

$y(0) = 1$; $3 - 1 = 0 + k$; $k = 2$; $3y - y^3 = x^3 + 2$ (show picture).

(We have an implicit representation of the solution). Hard to interpret.

Stud. Ex: Solve $\frac{dy}{dx} = \frac{3x^2+4x+2}{2y-2}$, $y(0) = -1$

Sol: $\int (2y-2)\,dy = \int (3x^2+4x+2)\,dx$; $y^2 - 2y = x^3 + 2x^2 + 2x + c$

$y(0) = -1$, $1 + 2 = c \Rightarrow c = 3$; $y^2 - 2y + 1 = x^3 + 2x^2 + 2x + 3$

$(y-1)^2 = x^3 + 2x^2 + 2x + 4$, $y = 1 \pm \sqrt{x^3 + 2x^2 + 2x + 4}$

$y = 1 - \sqrt{(x^2+2)(x+2)}$, Valid for $x > -2$. (Show DF, interpret). At $x = -2$ $dy/dx = \infty$.

Ex: Solve $\frac{dy}{dx} = \frac{4x - x^3}{4 + y^3}$, $y(0) = 1$. ; $\int (4+y^3)\,dy = \int (4x - x^3)\,dx$

$4y + \frac{y^4}{4} = 2x^2 - \frac{x^4}{4} + c$; $y^4 + 16y + x^4 - 8x^2 = c$ $\quad c = 17$

$y^4 + 16y + x^4 - 8x^2 - 17 = 0$ (Show DF, interpret).

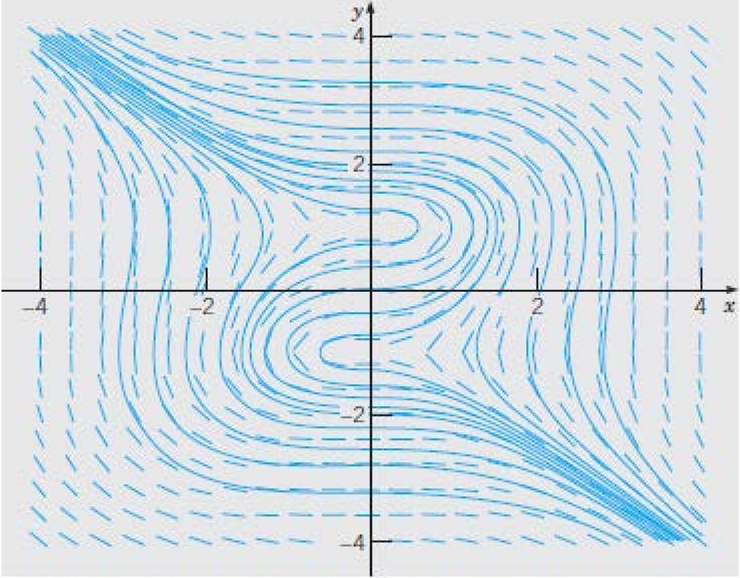

FIGURE 2.2.1 Direction field and integral curves of $y' = x^2/(1 - y^2)$.

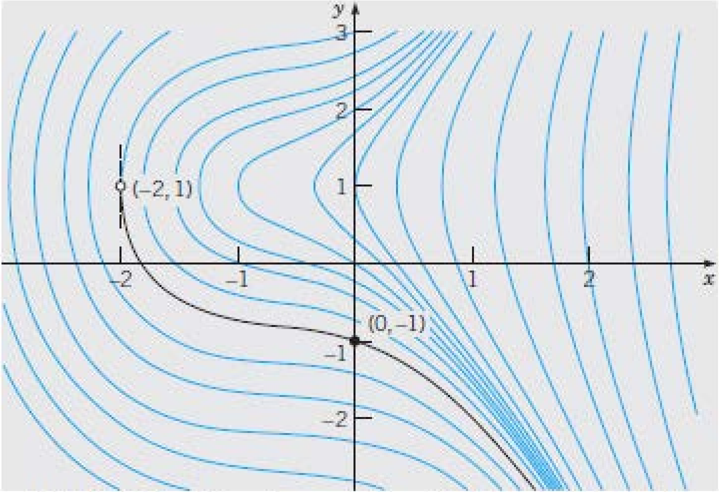

FIGURE 2.2.2 Integral curves of $y' = (3x^2 + 4x + 2)/2(y - 1)$; the solution satisfying $y(0) = -1$ is shown in black and is valid for $x > -2$.

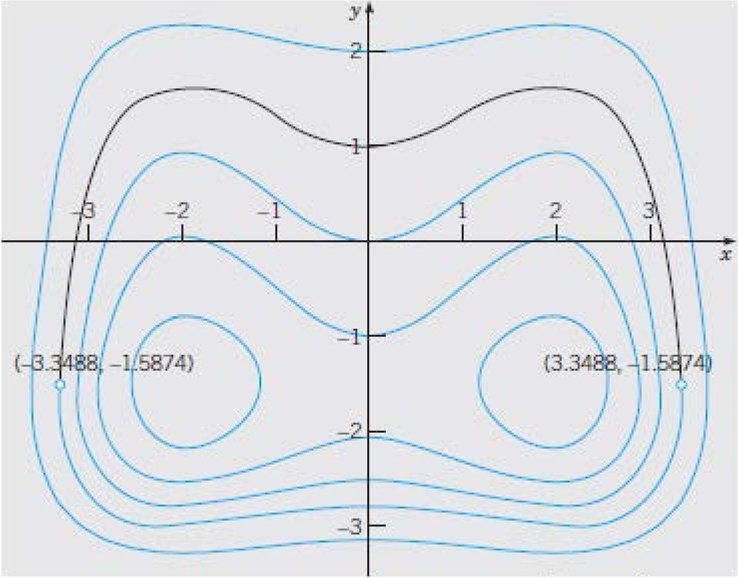

FIGURE 2.2.3 Integral curves of $y' = (4x - x^3)/(4 + y^3)$. The solution passing through $(0, 1)$ is shown by the black curve.