## Differential Equations and Dynamic Systems Modelling

**Course Code:** 201-HTL-05

**Ponderation:** 3-2-3

**Prerequisites:** 201-NYA and 201-NYB

**Course Description:**

This course is an introduction to the study of Differential Equations (DE). Particular emphasis will be placed throughout the course on using differential equations to model a wide variety of real-world problems involving dynamic systems, that is, systems which change and evolve with time. The modeling of such systems in order to study and manage complex systems is a methodology being used with increasing frequency in the engineering and social sciences. Dynamic systems modelling is used in engineering design studies, in ecological management, in urban planning, and in business management, to name just a few fields of application.

**Learning Outcomes:**

At the end of this course the student will be able to carry out the steps inherent to building mathematical models for various real-world physical systems. The student will be able to:

1. Create a mathematical model (involving one or more differential equations) which reasonably describes the behaviour of the system.

2. Find the mathematical solution to this idealized mathematical model by solving the differential equation(s), using appropriate methods.

3. Interpret the mathematical model (including the running computer simulations of the model if appropriate) to ensure that it adequately describes the behaviour of the physical system under study.

4. If the model does not adequately describe the behaviour of the system, the student will return to step 1 and revise the model.

   If the model is adequate in its prediction of system behaviour, using the model and experimenting with different parameter values for the system, the student will determine what changes should be made in the engineering design (or in the ecological, urban or business system) in order to optimize the behaviour of the system.

**Learning Activities:**

This course will refer to concepts seen in the students' CEGEP math courses, such as in Calculus and Linear Algebra, embodying concepts and principles encountered in physics, chemistry and biology courses. Students will be encouraged to ponder and analyse the behaviour of physical systems by working on projects. These projects may involve working in groups inside and outside of class. They will often involve working with the computer using MATLAB or Simulink software.

A short and very incomplete list of systems that we could model in this course might include:

- Population dynamics (single species, predator-prey, competing species)
- Medical dynamics (flu epidemics, drug ingestion/metabolism)
- Mechanical systems (pendulum, mass and spring, celestial mechanics)
- Oscillators (biological, physical, chemical)
- Electric circuits (with resistors, inductors, and capacitors: RLC circuits)
- Ecological systems (contamination of water bodies by pollution, Algae blooms in ocean environments)
- Social systems (urban development and urban decay models, heroin addiction and its impact on a community)

OBJECTIVE OOUV

**Statement of the competency**

STANDARD

To apply appropriate methods to solve or analyse the solution to differential equations (DE) encountered in the course, including the use of analytical methods (manipulation of formulae), graphical analysis of equations, and numerical techniques of solution (including computer simulations).
To apply the experimental method in mathematical modelling.

**Elements**

The student should be able

1. To use appropriate techniques in writing and solving first order DE and

   1.1 to model simple physical systems using DE.
   1.2 to solve equations using analytical techniques: separation of variables, linear DE.
   1.3 to analyse DE solutions using quantitative methods: slope fields.
   1.4 to solve DE using numerical techniques: Euler's method.
   1.5 to solve DE by change of variable.

2. To solve systems of first order DE using appropriate techniques, which include

   2.1 the modelling of physical problems using systems of DE.
   2.2 the use of analytical methods for special systems.
   2.3 the use of qualitative techniques: slope fields.
   2.4 using numerical methods: Euler's method for systems
   2.5 the use of linear methods when solving linear systems of DE.

3. To solve higher order DE using Laplace Transforms and

   3.1 to model physical systems using Laplace Transforms.
   3.2 to apply the technique to solve DE having discontinuous functions and/or impulse forcing functions present.

**Performance criteria**

Proper use of concepts, physical laws, and principles in creating mathematical models for physical systems.

Rigorous application of analytical techniques of solution, when that method is appropriate

Correct representations, in drawing or graph, when using qualitative methods of to describe the solutions of a DE.

Observance of good practice when using numerical techniques; recognition of the limitations inherent to the use of computer generated solutions to DE, and the approximate nature of the solution.

Justification of the approach used to solve a DE.

Assessment of the plausibility of the results obtained in modelling physical systems.

Observance of correct experimental method when simulating a physical system, and when studying a physical system to determine changes in system behaviour resulting from varying the parameter values.

The use of effective and clear communication, both in writing and orally, when presenting the results obtained in the work on projects.