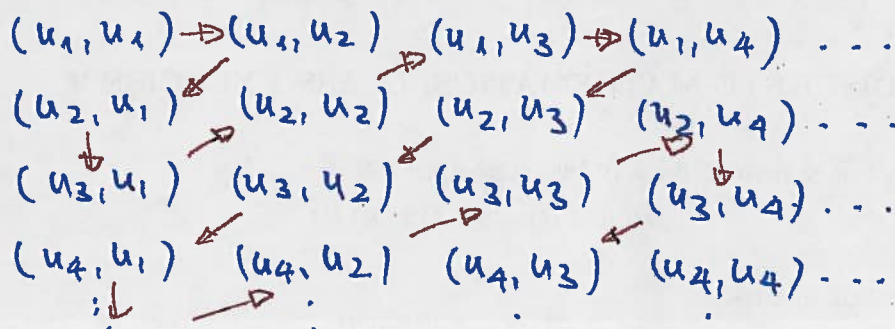


Disc Math - H16 - Clex 10 - Solutions

① We know that \mathbb{Z} is countable. Let $\{u_1, u_2, \dots\}$ be an enumeration of \mathbb{Z} . Arrange the elements of \mathbb{Z}^2 in a corner



Following the arrows gives an enumeration of \mathbb{Z}^2 .

② Consider the following mapping from $\mathcal{P}(S)$ to 2^S : for a subset $A \in \mathcal{P}(S)$ the function $f_A: S \rightarrow 2 = \{0, 1\}$ is

$$f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

$A \rightarrow f_A$ is clearly a bijection from $\mathcal{P}(S)$ to 2^S .

③ The new bijection is

$$h(1) = \mathbb{Z}; \quad h(u) = g(u-1), \quad u \geq 2$$

④ Let the Hilbert Hotel with countably infinitely many rooms be full and let countably infinitely many busses each with countably infinitely many new guests arrive. Move the preexisting guests to rooms 1, 3, 6, 10, 15, 21, 28, ... creating "gaps" of empty rooms of lengths 1, 2, 3, 4, 5, ... Accommodate the passengers from the 1st bus by putting them in the first available room in a gap (one per gap) starting with the 1st gap; accommodate the passengers from the second bus by putting them in the 1st available room in each gap (starting with the 2nd gap), etc. The first dozen rooms have the following guests



All guests (old and new) will be accommodated establishing a bijection between a countably infinite union of countably infinite sets and a countable set.