Disc Math - Clex 11 - Solutions

(1) i) $6 \times 6 = 36$, ii) $3 \times 3 = 9$

iii) To satisfy the condition the gray die could show 2, 3, 4, 5, 6. We add the number of choices on the brown die for each of the outcomes:

$$1 + 2 + 3 + 4 + 5 = 15$$

iv) For each possible outcome of the gray die the brown die has to show a number of the opposite parity. We add these possibilities:

$$3 + 3 + 3 + 3 + 3 + 3 = 18$$

(2) There is no need to consider the empty subset or the subset of all 6 numbers since these two subsets clearly cannot have sum equal to any other. Thus we have $2^6 - 2 = 62$ subsets. The maximum sum is $14 + 13 + 12 + 11 + 10 = 60$; the minimum sum is 1. 62 subsets, 60 possible sums $\Rightarrow$ at least two subsets have the same sum by the Pigeonhole Principle.

(3) There are $(10 \cdot 9 \cdot 8 \cdot 7)/4! = 210$ ways to select 4 integers out of 10 without order. The largest sum is $50 + 49 + 48 + 47 = 194$; the smallest sum is $1 + 2 + 3 + 4 = 10$. The number of possible sums is $194 - 10 + 1 = 185$. Choices of a set of 4 integers $= 210 > 185$ possible sums. By the Pigeonwhole Principle at least two selections have the same sum.

(4) Selecting characters in the string from left to right and not allowing the next character to be the same as the previous one we have

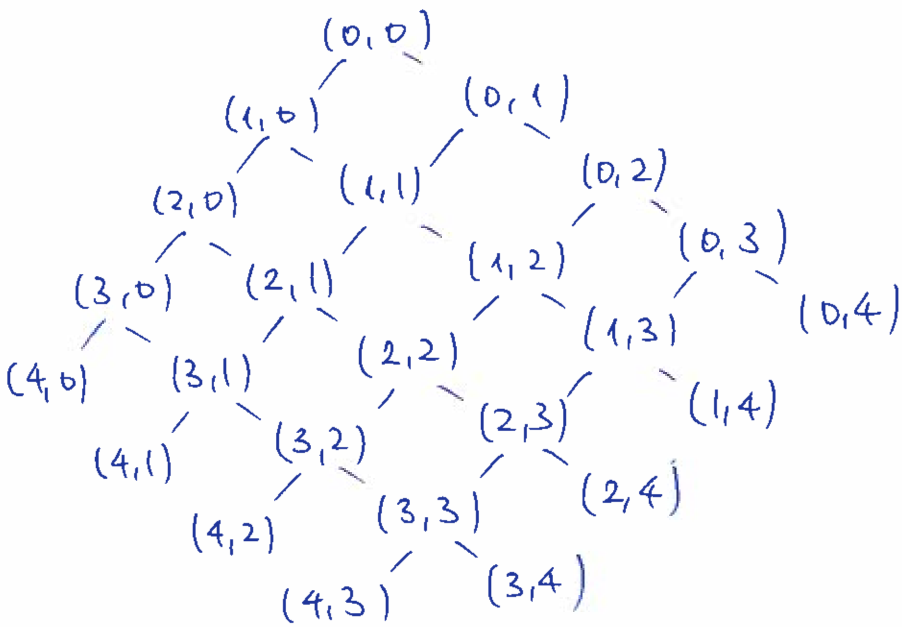
$$4 \cdot 3 \cdot 3 \cdot \ldots \cdot 3 = 4 \cdot 3^{n-1}$$

choices. The number of strings with at least one adjacent pair of characters that are the same is:

$$4^n - 4 \cdot 3^{n-1} = 4(4^{n-1} - 3^{n-1})$$

Disc Math - Clex II - Solutions

(5) We will denote by (i,j) the situation where Liberty has won i game and Lynx has won j games.



Liberty wins in 4 games $\rightarrow$ 1 way

Liberty wins in 5 games $\rightarrow$ 4 ways (Must win 3 in the first 4).

Liberty wins in 6 games $\rightarrow$ Must win 3 in the first 5 games $\rightarrow$

$(5\cdot 4\cdot 3)/3! = 10$

Liberty wins in 7 games $\rightarrow$ Must win 3 in the first 6 games $\rightarrow$

$(6\cdot 5\cdot 4)/3! = 20$

Liberty wins: $1+4+10+20 = 35$; Total number of scenarios $2\times 35 = 70$

(6) C_1 - divisible by 2^2; C_2 - divisible by 3^2; C_3 - divisible by 5^2

C_4 - divisible by 7^2; C_5 - divisible by 11^2

$|C_1| = \lfloor 150/2^2 \rfloor = 37$, $|C_2| = \lfloor 150/3^2 \rfloor = 16$, $|C_3| = \lfloor 150/5^2 \rfloor = 6$, $|C_4| = \lfloor 150/7^2 \rfloor = 3$

$|C_5| = \lfloor 150/11^2 \rfloor = 1$, $|C_1 \cap C_2| = \lfloor 150/2^2\cdot 3^2 \rfloor = 4$, $|C_1 \cap C_3| = \lfloor 150/2^2\cdot 5^2 \rfloor = 1$

$|C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5| = |C_1| + |C_2| + |C_3| + |C_4| + |C_5| - |C_1 \cap C_2| - |C_1 \cap C_3| =$

$= 37 + 16 + 6 + 3 + 1 - 4 - 1 = 58$

$|\text{Square-free}| = 150 - 58 = 92$