

## DISCRETE MATHEMATICS, CLASS EXERCISE 11

- (1) You roll two dice, one gray and one brown.
  - i) How many different outcomes are possible?
  - ii) How many rolls result in both dice showing an odd number?
  - iii) How many rolls where the number on the gray die is larger than the number on the brown die?
  - iv) How many rolls where the sum of the two numbers is odd?
- (2) Let  $A$  be a set of six distinct positive integers each of which is less than 15. Show that there must be two distinct subsets of  $A$  whose elements when added up give the same sum.
- (3) Let  $S$  be a set of ten integers chosen from 1 through 50. Show that the set contains at least two different (but not necessarily disjoint) subsets of four integers that add up to the same number. (For instance, if the ten numbers are  $\{3, 8, 9, 18, 24, 34, 35, 41, 44, 50\}$ , the subsets can be taken to be  $\{8, 24, 34, 35\}$  and  $\{9, 18, 24, 50\}$ . The numbers in both of these add up to 101.)
- (4) Consider strings of length  $n$  over the set  $\{a, b, c, d\}$ . How many such strings contain at least one pair of adjacent characters that are the same?
- (5) Draw a decision tree to find the number of ways the WNBA playoffs can happen, when the winner is the first team to win 4 out of 7. The two teams in the finals are Liberty and Lynx.
- (6) How many integers between 1 and 150 (inclusive) are square free? An integer,  $n$ , is called square free if it does not have a divisor of the form  $k^2$  where  $k$  is an integer.