

Disc Math - Clex 12 - Solutions.

$$1) a) p = \frac{{}^{13}C_1 + {}^{13}C_3 \cdot {}^{12}C_2 \cdot (4C_1)^2}{52C_5} = \frac{54912}{2598960} = 0.02113$$

$$b) p = \frac{1}{52C_5} [{}^{12}C_3 \cdot 40C_2 + 4C_1 \cdot {}^{13}C_5 - 4C_1 \cdot {}^{13}C_3 \cdot 10C_2] =$$

$$= \frac{176568}{2598960} = 0.06794$$

$$2) {}^{11}C_4 \cdot 1 + {}^{11}C_4 \cdot 1 - 10C_3 \cdot 1 = 330 + 330 - 120 = 540$$

$$3) \frac{22!}{5!5!6!6!} = 1.506 \times 10^{11}$$

$$4) - \frac{7!}{2!2!3!} = -210$$

5) combinations with repetitions:

$$a) r=6, u=3; u+r-1C_r = 8C_6 = 28$$

$$b) r=6, u=2; 7C_6 = 7$$

$$c) r=3, u=3; 5C_3 = 10$$

$$6) 6^u = (2+4)^u = \sum_{k=0}^u {}^u C_k 2^{u-k} 4^k = \sum_{k=0}^u {}^u C_k 2^{u-k+2k} = \sum_{k=0}^u {}^u C_k 2^{u+k}$$

$$\textcircled{7} \text{ LHS} = {}^u C_r \cdot r^k = \frac{u!}{\cancel{r!} (u-r)!} \cdot \frac{\cancel{r!}}{k! (r-k)!} = \frac{u!}{k! (u-r)! (r-k)!}$$

$$\text{RHS} = {}^u C_k \cdot {}^{u-k} C_{r-k} = \frac{u!}{k! \cancel{(u-k)!}} \cdot \frac{\cancel{(u-k)!}}{(r-k)! (u-r)!} \cdot \frac{u!}{k! (u-r)! (r-k)!} = \text{LHS}$$

$$\textcircled{8} (x+y+z)^4 = \sum_{i=0}^4 \sum_{j=0}^{4-i} \frac{4!}{(4-i-j)! i! j!} x^{4-i-j} y^i z^j =$$

$$= x^4 + 4x^3z + 6x^2z^2 + 4xz^3 + z^4 + 4x^3y + 12x^2yz + 12xy^2z +$$

$$+ 4yz^3 + 6x^2y^2 + 12xy^2z + 6y^2z^2 + 4xy^3 + 4y^3z + y^4$$