

DISCRETE MATHEMATICS, CLASS EXERCISE 12

- (1) a) A poker hand that contains three cards of the same rank, plus two cards which are not of this rank nor the same as each other is called "three of a kind". Compute the probability that a randomly selected poker hand is "three of a kind".
b) Compute the probability that a randomly selected poker hand contains precisely 3 face cards or 5 cards of the same suit.
- (2) A student must select 5 classes for next semester from among 12 classes, but one of the classes must be either Canadian history or French literature. How many sets of classes are possible?
- (3) A programming team leader needs to split his 22 programmers into 4 groups for work on a project. How many ways are there to do this if two of the groups will have 5 programmers and the other two groups will consist of 6 programmers.
- (4) What is the coefficient of $x^2y^2z^3$ in the expansion of $(x + y - z)^7$?
- (5) A hardware shipping order contains 6 items, where each item is either a screwdriver, a hammer, or a drill.
a) How many different shipping orders are possible?
b) How many different orders are possible if no drills are shipped?
c) How many different shipping orders are possible if each order must contain at least one screwdriver, one hammer and one drill?
- (6) Use the statement of the Binomial Theorem to prove the identity

$$\sum_{i=0}^n {}_n C_i 2^{n+i} = 6^n$$

- (7) Prove the identity

$${}_n C_r \cdot {}_r C_k = {}_n C_k \cdot {}_{n-k} C_{r-k} \quad \text{for } k \leq r \leq n$$

- (8) Use the binomial theorem (more than once) to expand $(x + y + z)^4$.