DISCRETE MATHEMATICS, CLASS EXERCISE 13

(1) If R and S are transitive relations on a set A prove or disprove that the relation $R \cup S$ is also transitive.

(2) Let D be the relation defined on $\mathbb{Z}$ as follows: $\forall m, n \in \mathbb{Z}$

$$m \; D \; n \iff 3 | (m^2 - n^2)$$

Prove that this relation is an equivalence relation and describe the distinct equivalence classes of this relation.

(3) Define a relation S on $\mathbb{R}$ as follows: For all real numbers x and y, $x \; S \; y \iff x - y \in \mathbb{Q}$. Determine whether this relation is reflexive, symmetric, transitive, or none of these.

(4) Let (S, ρ) and (T, σ) be two partially ordered sets. A relation μ on $S \times T$ is defined by $(s_1, t_1)\mu(s_2, t_2) \leftrightarrow \{s_1 \rho s_2 \;\; \wedge \;\; t_1 \sigma t_2\}$. Show that μ is a partial ordering on $S \times T$.

(5) Draw the Hasse diagram of the partially ordered set $\mathcal{P}(\{0, 1, 2\})$. Does this poset have any greatest, least, maximal and/or minimal elements?

(6) Let $S = \mathbb{N} \times \mathbb{N}$ and let ρ be the binary relation on S defined by $(x, y)\rho(z, w) \leftrightarrow x + y = z + w$. Show that ρ is an equivalence relation on S and describe its equivalence classes.

(7) Let ρ and σ be partial orders on the same set A. Prove or disprove that $\rho \cap \sigma$ is a partial order on A.

(8) Solve the equation $22x + 14 = 8 - 29x$ in $\mathbb{Z}_{37}$.