

① False. Let  $S = \{a\}$ ,  $T = \{b, c\}$ ,  $U = \{d\}$ . Let  $f(a) = b$ ,  $g(b) = g(c) = d$ . Then  $(g \circ f)(a) = d$  is a bijection, but neither  $f$  nor  $g$  is a bijection.

$$\begin{aligned} \textcircled{2} \quad & \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & \dots \\ 1 & 8 & 3 & 2 & 5 & 4 & 7 & 6 & 9 & 10 & 11 & 12 & 13 & \dots \\ 1 & 8 & 3 & 2 & 5 & 10 & 7 & 6 & 9 & 11 & 4 & 12 & 13 & \dots \\ 1 & 8 & 3 & 11 & 5 & 10 & 7 & 2 & 9 & 13 & 4 & 12 & 6 & \dots \end{array} \\ & \quad \quad \quad \downarrow (2, 8, 6, 4) \\ & \quad \quad \quad \downarrow (10, 11, 4) \\ & \quad \quad \quad \downarrow (2, 11, 13, 6) \\ & = (2, 8) \circ (4, 11) \circ (6, 10, 13) \end{aligned}$$

③ a) Reflexivity  $f(x) = f(x) \checkmark$

Symmetry  $f(x) = f(y) \Rightarrow f(y) = f(x) \checkmark$

Transitivity  $f(x) = f(y) \wedge f(y) = f(z) \Rightarrow f(x) = f(z) \checkmark$

b) If  $f$  is an injection, no distinct elements of  $S$  have the same image under  $f$ . The equivalence classes are the elements of  $S$ .

c) The equivalence classes are the points of the interval  $[-\pi/2, \pi/2]$ .

$$\textcircled{4} \quad 4^4 - {}_4C_1 \cdot 3^4 + {}_4C_2 \cdot 2^4 - {}_4C_3 \cdot 1^4 = 24$$

Any surjection  $S \rightarrow S$  will cover the elements  $a, b, c, d$  of  $S$  and will therefore be a permutation of these elements, i.e. a bijection.

$$\textcircled{5} \quad \#(A \rightarrow \mathbb{Z}) = 2^u, \quad \#(\mathbb{Z} \rightarrow A) = u^2$$

For large  $u$ ,  $\#(A \rightarrow \mathbb{Z}) \gg \#(\mathbb{Z} \rightarrow A)$

⑥  $f$  is 1-1 since if  $s \neq t$ ,  $s, t \in A^* \Rightarrow xs \neq xt$ .

$f$  is not onto since its range does not contain the strings in  $A^*$  that start with  $y$ .

$$\textcircled{7} \quad \text{a) } 2^{4 \cdot 3} = 4096 \text{ relations; } \text{b) } 3^4 = 81 \text{ functions}$$

c) Injective: none since  $|S| > |T|$

Bijjective: none since  $|S| > |T|$

$$\text{Surjective: } 3^4 - {}_3C_1(3-1)^4 + {}_3C_2(3-2)^4 = 81 - 48 + 3 = 36$$