DISCRETE MATHEMATICS, CLASS EXERCISE 14

(1) Let $f : S \to T$ and $g : T \to U$ be functions. Prove or disprove that $g \circ f$ is a bijection iff f and g are bijections.

(2) Find the composition of the following cycles representing permutations on $\mathbb{N}$. Write your answer as a union of one or more disjoint cycles.

$$(2, 11, 13, 6) \circ (10, 11, 4) \circ (2, 8, 6, 4)$$

(3) Let $f : S \to T$ be a function. Define a binary relation ρ on S by $x\rho y \leftrightarrow f(x) = f(y)$.
a) Prove that ρ is an equivalence relation.
b) What could be said about the equivalence classes if f is an injection?
c) For $S = T = \mathbb{R}$ and $f(x) = \sin x$, describe the equivalence classes of the equivalence relation on $\mathbb{R}$ defined by this function.

(4) Let $S = \{a, b, c, d\}$. How many surjective functions are there from S to itself? Prove or disprove that any surjection $S \to S$ is necessarily also a bijection.

(5) Let A be a finite set of cardinality n and let $\mathbf{2} = \{0, 1\}$ be the two element set. Compare the cardinalities of the two sets of functions $A \to \mathbf{2}$ and $\mathbf{2} \to A$. Which one is larger for large n.

(6) Let $A = \{x, y\}$ and let A^* be the set of all strings of finite length made up of symbols from A. A function $f : A^* \to A^*$ is defined as follows: for $s \in A^*, f(s) = xs$ (the single character string x followed by s). Is f one-to-one? Prove or disprove. Is f onto? Prove-or-disprove.

(7) Let $S = \{a, b, c, d\}$ and $T = \{1, 2, a\}$.
a) How many relations are there from S to T?
b) How many functions are there from S to T?
c) How many injective, surjective and bijective functions are there from S to T?