DISCRETE MATHEMATICS, CLASS EXERCISE 15

(1) Draw all nonisomorphic simple graphs with 3 nodes. Make sure you do not include different representations of the same graph on your list.

(2) Recall that K_n denotes the complete (simple) graph on n vertices.
a) Prove that for $n \geq 1$, the number of edges of K_n is $n(n-1)/2$.
b) Argue that the number of edges of a simple graph with n vertices is less than or equal to $n(n-1)/2$.
c) Is there a simple graph with twice as many edges as vertices? Explain.

(3) Either draw a graph with the given specification or explain why no such graph exists:
a) Graph, connected, 6 vertices, 5 edges, has a cycle.
b) Forest, 6 vertices, total degree 12.
c) Full binary tree 7 vertices, 4 leaves.
d) Binary tree, height 3, 8 internal vertices.

(4) Prove that a connected planar graph (not necessarily simple) obeys Euler's formula when drawn in its planar form.

(5) Prove that if G is a tree, u and v are two distinct nonadjacent nodes in G, then adding an arc $\{u, v\}$ to G will form a graph with exactly one cycle.

(6) Consider the binary relation $\rho = \{(2,1),(2,3),(3,4),(4,1)\}$ on the set $\{1,2,3,4\}$.
a) Draw the associated directed graph and the adjacency matrix.
b) Determine the transitive closure of ρ by computing the reachability matrix (show details).

(7) Prove that a simple connected graph with m arcs has at most $m+1$ nodes.