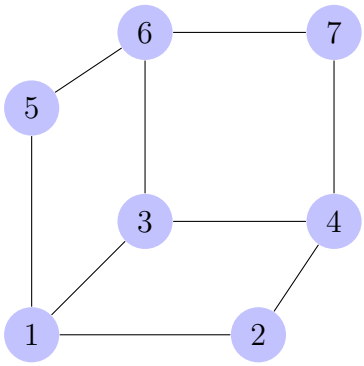
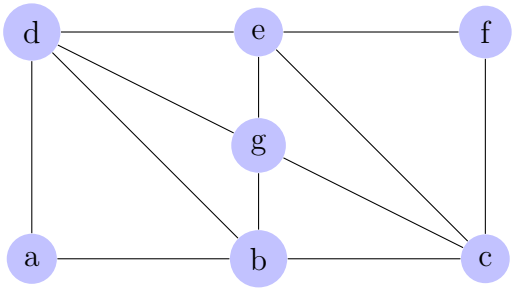
DISCRETE MATHEMATICS, CLASS EXERCISE 16

(1) Determine whether each of the following two graphs has an Euler path or a Hamiltonian circuit. It such paths (circuits) exist list their vertices in order.





(2) A directed graph $\mathcal{G}$ with vertices $\{a, b, c, d\}$ has adjacency matrix A. Draw the graph and compute the reachablity matrix R for this graph.

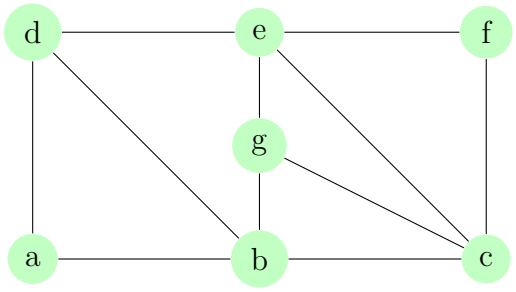
$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

(3) Let G be a directed graph, possibly with parallel arcs, and let A be its adjacency matrix (it may be not Boolean).
i) Prove that the i, j entry of the matrix A^2 is the number of paths of length 2 from node i to node j.
ii) Prove that the i, j entry of the matrix A^n is the number of paths of length n from node i to node j.

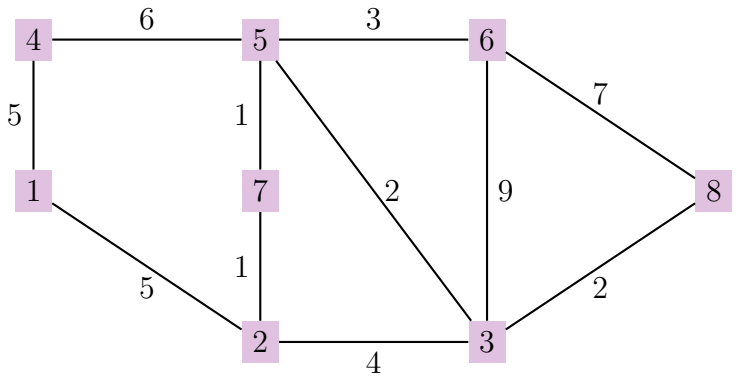
(4) The definition of an Euler path extends to directed graphs. We must now keep track of the total number of arcs coming into a node, its *in-degree* and the total number of arcs leaving a node, its *out-degree*. Describe the two conditions on a connected directed graph, either of which would guarantee the existence of an Euler path.

(5) Prove that a Hamilonian circuit exists in a connected graph where every node has degree 2.

(6) For the following graph, while using alphabetical order and describing the steps in the algorithm you are using
i) Write the nodes in a breadth-first search.
ii) Write the nodes in a depth-first search.



(7) For the graph below, while describing every step in the algorithm you are using,
i) Find a shortest path from node 1 to node 8. Please provide the details of Dijkstra's algorithm.
ii) Find a minimal spanning tree. Please provide the details of Prim's algorithm.



(8) Recall that $K_{m,n}$ denotes the bipartite, complete graph with $m+n$ nodes.
a) For what values of m and n does an Euler path exist in $K_{m,n}$?

b) For what values of m and n does a Hamiltonian circuit exist in $K_{m,n}$?

(9) Let a be the arc of lowest weight in a weighted graph. Show that a must be an arc in any minimal spanning tree.

(10) Describe how the depth-first search algorithm can be used in a connected graph to detect the presence of cycles in the graph. Draw a graph illustarting your reasoning.