Discrete Math - H16 - Clex3 - Solutions

(1) $J \vee L \to C,\ T',\ C \to T \vdash J'$

$$\dfrac{C \to T,\ T'}{C'}\ \text{MT}$$

$$\dfrac{C' \qquad J \vee L \to C}{(J \vee L)'}\ \text{MT}$$

$$\dfrac{(J \vee L)'}{J' \wedge L'}\ \text{DeMorgan}$$

$$\dfrac{J' \wedge L'}{J'}\ \text{simpl.}$$

(2) i) Statement: $\forall n \in \mathbb{Z},\ 1/n \in \mathbb{Q}$ False
Negation: $\exists n \in \mathbb{Z}$ s.t. $1/n \notin \mathbb{Q}$ True } $n = 0$

ii) Statement: $\exists x, y \in \mathbb{Q}$ s.t. $x + y \in \mathbb{Z} \wedge xy \notin \mathbb{Z}$ True
Negation: $\forall x, y \in \mathbb{Q},\ x + y \notin \mathbb{Z} \vee xy \in \mathbb{Z}$ False } $x = y = 1/2$

(3) Statement: $\forall x, \forall y, \exists z$ s.t. $((x<y) \to ((z>x) \wedge (z<y)))$ False
Negation: $\exists x$ s.t. $\exists y$ s.t. $\forall z,\ (x<y) \wedge [(z \le x) \vee (z \ge y)]$ True } $x = 0,\ y = 1.$

(4) $\{\forall x \in R,\ S(x) \vee P(x)\},\ P(A) \vdash \sim S(A)$

$$\dfrac{\forall x \in R,\ S(x) \vee P(x)}{S(A) \vee P(A)}\ \text{UI}$$

$$\dfrac{S(A) \vee P(A)}{\sim S(A) \to P(A)}\ \text{impl.}$$

$$\dfrac{\sim S(A) \to P(A) \qquad P(A)}{\sim S(A)}$$

! Converse error.
! Invalid argument.

A could be both a square and a parallelogram.

Discrete Math - H16 - Clex 3 - solutions

(5)

$\forall x, P(x) \rightarrow \exists y$ s.t. $Q(x,y)$ — ui

(T.H.) $P(x)$ $\quad$ $P(x) \rightarrow \exists y$ s.t. $Q(x,y)$ — MP

$\exists y$ s.t. $Q(x,y)$ — ei

$Q(x,a)$. — Discharge Temporary Hyp.

$P(x) \rightarrow Q(x,a)$ — eg

$\exists y$ s.t. $P(x) \rightarrow Q(x,y)$ — ug

$\forall x, \exists y$ s.t. $P(x) \rightarrow Q(x,y)$.

Note: Shorter derivation is based on sliding the quantifier $\exists y$ past the predicate $P(x)$.

(6) $\{\exists x$ s.t. $\forall y, C(x) \wedge S(x,y)\}, \{\forall x, y, S(x,y) \rightarrow B(x,y)\} \vdash \exists x$ s.t. $\forall y, C(x) \wedge B(x,y)$

$\exists x$ s.t. $\forall y, C(x) \wedge S(x,y)$ — ei

$\forall y, C(a) \wedge S(a,y)$ — ui

$C(a) \wedge S(a,y)$ — simp

$\forall x, \forall y, S(x,y) \rightarrow B(x,y)$ — ui

$\forall y, S(a,y) \rightarrow B(a,y)$ — ui

$C(a)$ $\quad$ $S(a,y)$ $\quad$ $S(a,y) \rightarrow B(a,y)$ — MP

$C(a)$ $\quad$ $B(a,y)$

$C(a) \wedge B(a,y)$ — ug

$\forall y, C(a) \wedge B(a,y)$ — eg

$\exists x$ s.t. $\forall y, C(x) \wedge B(x,y)$