DISCRETE MATHEMATICS, CLASS EXERCISE 3

(1) Write the following argument using the propositional letters J, L, C, T. Then using propositional logic (and not a truth table) show that the qrgument is valid.

"If Natasha took the jewelry or Mrs. Krasovska lied, then a crime was committed. Mr. Krasovski was not in town. If a crime was committed, then Mr. Krasovski was in town. Therefore, Nathasha did not take the jewelry."

(2) Rewrite the following statements formally. Use variables and include the quantifiers $\forall$ and $\exists$ in your answers. Next, for both i) and ii) write the negation, and argue that the statement is true or the corresponding negation is true. Justify your answers.

i) For every integer, its reciprocal is a rational number.

ii) There are rational numbers x and y such that $x + y \in \mathbb{Z}$ and $xy \notin \mathbb{Z}$.

(3) Write the negation of the proposition

$$\forall x, \forall y, \exists z \text{ s. t. } ((x < y) \to ((z > x) \land (z < y))).$$

If the domain of discourse is $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$ is the original proposition true or is the negation true? Justify your answer.

(4) Is the following arguments valid or invalid? Justify your answer.

"Every rhombus is a square or a parallelogram. Rhombus A is a parallelogram. Therefore, rhombus A is not a square."

(5) Using predicate logic prove that the following argument is valid.

$$(\forall x, P(x) \to \exists y \text{ s.t. } Q(x, y)) \vdash (\forall x, \exists y \text{ s.t. } P(x) \to Q(x, y))$$

(6) Use predicate logic to prove that the following argument is valid. Use the predicate symbols $C(x), S(x, y), B(x, y)$.

"There is some computer scientist who is smarter than everyone. Anyone who is smarter than anyone else has better chance of solving P vs. NP than anyone else does. Therefore, there is a computer scientist who has a better chance of solving P vs. NP than anyone else."