# Disc Math - H16 - Clex 5 solutions

① Base case $n=2$: $2^2 = 4 < (2+1)! = 6$. Assume $2^k < (k+1)!$ Then

$2^{k+1} = 2 \cdot 2^k < 2 \cdot (k+1)! < (k+2)(k+1)! = (k+2)!$

② Base case $n=1$: $1 \cdot 2 = \frac{1 \cdot 2 \cdot 3}{3}$ V Assume $\sum_{i=1}^{k} i(i+1) = \frac{k(k+1)(k+2)}{3}$

$$\sum_{i=1}^{k+1} i(i+1) = \sum_{i=1}^{k} i(i+1) + (k+1)(k+2) = \frac{k(k+1)(k+2)}{3} + (k+1)(k+2) =$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

③ Base case $n=1$: $7^2 + 16 - 1 = 64$ V Assume $7^{2k} + 16k - 1 = 64p, p \in$

$$7^{2(k+1)} + 16(k+1) - 1 = 49 \cdot 7^{2k} + 16k + 15 = 49(64p - 16k + 1) + 16k + 15$$

$$= 64(49p - 12k + 1)$$

④ Base cases: $f(1) = 2 \cdot 5^0 + 6 = 8$, $f(2) = 2 \cdot 5^1 + 6 = 16$ V

Assume $f(i) = 2 \cdot 5^{i-1} + 6$, for all $1 \le i \le k$. Then

$$f(k+1) = 6f(k) - 5f(k-1) = 6(2 \cdot 5^{k-1} + 6) - 5(2 \cdot 5^{k-2} + 6) =$$

$$= 12 \cdot 5^{k-1} + 36 - 2 \cdot 5^{k-1} - 30 = 2 \cdot 5^k + 6.$$

⑤ $792 = 4 \cdot 165 + 132$, $165 = 1 \cdot 132 + 33$, $132 = 4 \cdot 33 + 0$

$\gcd(792, 165) = 33$

$33 = 165 - 132 = 165 - (792 - 4 \cdot 165) = -792 + 5 \cdot 165$

⑥ False. Take $m=2, n=3$. Then $\gcd(m,n) = \gcd(2,3) = 1$.

However $\gcd(m, m+2n) = \gcd(2,8) = 2$.