DISCRETE MATHEMATICS, CLASS EXERCISE 5

(1) Prove by mathematical induction that $\forall n \in \mathbb{N}, n \geq 2$
$$2^n < (n+1)!$$

(2) Prove by mathematical induction that $\forall n \in \mathbb{N}$,
$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}.$$

(3) Prove that $7^{2n} + 16n - 1$ is divisible by 64 for every positive integer n.

(4) A sequence is defined recursively via $f(1) = 8$, $f(2) = 16$ and
$$f(n) = 6f(n-1) - 5f(n-2), \quad n \geq 3.$$
Prove that
$$f(n) = 2(5)^{n-1} + 6.$$

(5) Compute $\gcd(792, 165)$ and then write it as an integer combination of 792 and 165.

(6) Prove or disprove that for any positive integers m and n,
$$\gcd(m, n) = \gcd(m, m + 2n).$$