Disc Math - Clex 6 - Solutions

① Base case $n=0$. $13^0-6^0=0$ ✓. Assume $13^k-6^k=7t$. The

$13^{k+1}-6^{k+1}=13\cdot 13^k-6\cdot 6^k=13(7t+6^k)-6\cdot 6^k=7(13t+6^k$

② $211=1\cdot 158+53$, $158=2\cdot 53+52$, $53=1\cdot 52+1$, $52=52\cdot 1+$

$1=53-52=53-(158-2\cdot 53)=3(53)-158=3(211-158)-158=$

$=3(211)-4(158) \Rightarrow 158^{-1} \bmod 211 = -4 \bmod 211 = 207 \bmod 2$

$x=(207)(26) \bmod 211 = 107 \bmod 211$.

③ $117612=2^2\cdot 3^5\cdot 11^2$; $\varphi(117612)=2^1\cdot 3^4\cdot 11^1[1\cdot 2\cdot 10]=35640$

④ The numbers smaller than p^k which are not relatively prim to p^k are the multiples of p. There are p^k/p such multiple which are $\leq p^k$. Thus $\varphi(p^k)=p^k-p^k/p=p^k-p^{k-1}=p^{k-1}(p-1)=$

$=p^{k-1}\varphi(p)$

⑤ Let $r=p_1^{r_1}\dots p_k^{r_k}$, $s=q_1^{s_1}\dots q_j^{s_j}$ where the p's and the q's are distinct.

$\varphi(rs)=p_1^{r_1-1}\dots p_k^{r_k-1}q_1^{s_1-1}\dots q_j^{s_j-1}\varphi(p_1)\dots\varphi(p_k)\varphi(q_1)\dots\varphi(q_j)=$

$=p_1^{r_1-1}\dots p_k^{r_k-1}\varphi(p_1)\dots\varphi(p_k)\,q_1^{s_1-1}\dots q_j^{s_j-1}\varphi(q_1)\dots\varphi(q_j)=\varphi(r)\varphi(s)$.