

## DISCRETE MATHEMATICS, CLASS EXERCISE 6

- (1) Prove that  $13^n - 6^n$  is divisible by 7 for  $n \geq 0$ .
- (2) Solve the equation  $158x = 26 \pmod{211}$ .
- (3) Find the prime number factorization of 117612 and compute  $\varphi(n)$ .
- (4) For a prime number  $p$  and any positive integer  $k$ , prove that  $\varphi(p^k) = p^{k-1}\varphi(p)$  using the definition of the Euler  $\varphi$  function.
- (5) Prove that if  $r$  and  $s$  are relatively prime, then  $\varphi(rs) = \varphi(r)\varphi(s)$ .