Disc Math - H16 - Clex7 - Solutions 

① $\phi(r) = 22 \cdot 30 = 660$ , $s = 13$

$660 = 50 \cdot 13 + 10$ , $13 = 1 \cdot 10 + 3$ , $10 = 3 \cdot 3 + 1$ , $3 = 3 \cdot 1 + 0$

$1 = 10 - 3 \cdot 3 = 10 - 3(13 - 10) = -3(13) + 4(10) = -3(13) + 4(660 - 50 \cdot 13)$

$= 4(660) - 203(13)$

$13^{-1} \bmod 660 = -203 \bmod 660 = 457 \bmod 660$ $\quad t = 457$

$E = 5^{457} \bmod 713 = (5^{10})^{45} \cdot 5^7 \bmod 713 = (377)^{45} \cdot 408 \bmod 7$

$= (377^3)^{15} \cdot 408 \bmod 713 = (683^3)^5 \cdot 408 \bmod 731 = (94)^5 \cdot 408 \bmod$

$= 32 \cdot 408 \bmod 713 = 222 \bmod 713$ $\quad M = 222$

② $F(1) = 1$ , $F(2) = 1$ , $F(3) = 2$ , $F(4) = 3$ , $F(5) = 5$ , $F(6) = 8$

Base cases: $n = 1$ , $F(4) = 2F(2) + F(1) = 2 \cdot 1 + 1 = 3$ V

$F(5) = 2F(3) + F(2) = 2 \cdot 2 + 1 = 5$ V

Assume $F(i+3) = 2F(i+1) + F(i)$ , $1 \le i \le n$.

Prove $F(n+4) = 2F(n+2) + F(n+1)$. Indeed,

$F(n+4) = F(n+3) + F(n+2) = 2F(n+1) + F(n) + 2F(n) + F(n-1)$

$= 2[F(n+1) + F(n)] + [F(n) + F(n-1)] \underset{\uparrow}{=} 2F(n+2) + F(n+1)$

inductive assumpt.

③ Fibonacci ($n \in \mathbb{N}$)

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if n = 1 then
    return 1
elseif n = 2 then
    return 1
else
    return F(n-1) + F(n-2)
end if
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# Disc Math - H16 - Clex 7 - Solutions

4 a) 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, ...

b) Base cases: $L(2) = F(3) + F(1) = 2+1=3$

$L(3) = F(4) + F(2) = 3+1 = 4$

Assume $L(i) = F(i+1) + F(i-1)$, $2 \leq i \leq k$.

Prove $L(k+1) = F(k+2) + F(k)$.

$$L(k+1) = L(k) + L(k-1) \underset{\text{Ind. Assumpt.}}{=} F(k+1) + F(k-1) + F(k) + F(k-2) =$$

$$= \left[F(k+1) + F(k)\right] + \left[F(k-1) + F(k-2)\right] = F(k+2) + F(k).$$