DISCRETE MATHEMATICS, CLASS EXERCISE 8

(1) A sequence is defined recursively by : $C(k) = 3C(k-1) + 1, k \geq 2$ and $C(1) = 1$.
a) Use iteration to guess an explicit formula for this sequence.
b) Now use mathematical induction to verify the correctness of your formula.

(2) A video is turning viral according to the following model: One person sends the link to 3 friends, each of whom sends the link to to 3 friends, and so forth.
a) How many people have received the link after the 20'th iteration of this process, assuming no person receives the link more than once.
b) If there are 7 billion people on Earth, how many iterations will it take for the link to be send to every person according to this model?

(3) Solve the recurrence relation subject to the initial conditions

$$F(n) = 6F(n-1) - 5F(n-2),\ n \geq 3; \quad F(1) = 8,\ F(2) = 16.$$

(4) Solve the recurrence relation subject to the initial conditions

$$C(n) = 4C(n-1) - 4C(n-2), \quad C(1) = -2,\ C(2) = 12$$

(5) Argue that $f = \Theta(g)$ if $f(x) = 4x^2 + 3x + 5\ln x$ and $g(x) = x^2$.

(6) Give an argument (quantative) proving that $\log(3x^2) = \Theta(\log x)$.

(7) Argue that $(\ln x)^2 = o(\sqrt{x})$.