# Disc H16 - clex 9 - Solutions

① False! $A\cup B \in P(A\cup B)$, but $A\cup B \notin P(A) \wedge A\cup B \notin P(B)$

$\Rightarrow A\cup B \notin P(A)\cup P(B)$.

② $(A\setminus B)\setminus C = (A\cap B')\cap C' = A\cap(B'\cap C') = A\cap(C'\cap B') =$

$= (A\cap C')\cap B' = (A\setminus C)\setminus B$

③ A generic element of the LHS is of the form $(a,b)$ with $a\in A$, $b\in \bigcap_{i=1}^{n} B$ and so is a generic element of the RHS.

④ $((A\cap(B\cup C))\cap(A\setminus B)\cap(B\cup C') = ((A\cap(B\cup C))\cap(A\cap B')\cap(B\cup C')$

$\underset{\substack{\text{assoc.}\\ \text{comm.}}}{=} A\cap A\cap B'\cap(B\cup C)\cap(B\cup C') \underset{\substack{\text{Idemp.}\\ \text{distr.}}}{=} A\cap B'\cap[B\cup(C\cap C')] =$

$\underset{\text{compl.}}{=} A\cap B'\cap[B\cup\emptyset] \underset{\text{Ident.}}{=} A\cap B'\cap B \underset{\text{compl.}}{=} A\cap\emptyset \underset{\text{ident.}}{=} \emptyset$

⑤ False, here is a counterexample.

$A=\{a,b\}$, $B=\{b\}$, $C=\{c,d\}$, $D=\{d\}$

$(A\setminus B)\times(C\setminus D) = \{a\}\times\{c\} = \{(a,c)\}$

$(A\times C)\setminus(B\times D) = \{(a,c),(a,d),(b,c)\} \neq (A\setminus B)\times(C\setminus D)$

⑥ $(A\cap B)\setminus(B\cap C) \underset{\text{def}}{=} (A\cap B)\cap(B\cap C)' \underset{\text{DM}}{=} (A\cap B)\cap(B'\cup C') \underset{\text{distr.}}{=}$

$= (A\cap B\cap B')\cup(A\cap B\cap C') \underset{\text{compl.}}{=} \emptyset\cup(A\cap B\cap C') \underset{\text{id.}}{=} A\cap B\cap C' \underset{\text{def}}{=}$

$= (A\cap B)\setminus C$