DISCRETE MATHEMATICS, CLASS EXERCISE 9

(1) Prove or disprove, for any two sets A and B
$$\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$$

(2) Prove or disprove
$$(A \backslash B) \backslash C = (A \backslash C) \backslash B$$

(3) Prove that for all $n \geq 1$, if A and $B_1, B_2, \ldots$ are any sets, then
$$\cap_{i=1}^{n}(A \times B_i) = A \times (\cap_{i=1}^{n} B_i).$$

(4) Consider sets A, B and C. Simplify the following expression. Cite a Boolean algebra identity at every step.
$$((A \cap (B \cup C)) \cap (A \backslash B)) \cap (B \cup C^c).$$

(5) Prove or disprove that for any sets A, B, C and D
$$(A \backslash B) \times (C \backslash D) = (A \times C) \backslash (B \times D).$$

(6) Use Boolean algebra identities to prove or disprove the following formula for any sets A, B and C:
$$(A \cap B) \backslash (B \cap C) = (A \cap B) \backslash C.$$
Cite the relevant Boolean algebra identity at each step.