Disc Math - §13 - Counting

Principle: (Multiplication) If there are n_1 possible outcomes for a first event, n_2 for a second event, .., n_k outcomes for event "k", there are

$$n_1 \times n_2 \times \dots \times n_k$$

possible outcomes for the sequence of k events.

Ex: PIN of 4-digits $\rightarrow 10^4$ PIN's

PIN of 4-digits, no repetition $\rightarrow 10 \times 9 \times 8 \times 7 = 5040$ PIN's

Ex: Binary strings of length $n \rightarrow 2^n$

Subsets of a set of cardinality $n \rightarrow 2^n$

Ex: $\# A \times B = \# A \cdot \# B$.

Principle: (Addition). If A and B are disjoint events with n_1 and n_2 possible outcomes, respectively, then the total number of possible outcomes of the event A or B is $n_1 + n_2$. (Same for k disjoint events)

Ex: For dessert, there is a choice of 5 pies or 3 cakes $\rightarrow$ 8 choices of desert.

Ex: If $A \cap B = \emptyset$, $\#(A \cup B) = \# A + \# B$.

Also $\# A \setminus B = \# A - \# A \cap B$.

Ex: How many 4 digit PIN's begin with 1 or with 8?

$10^3 + 10^3 = 2 \times 10^3$.

Ex: How many 4 digit pins with at least one repetition:

$10^4 - 10 \cdot 9 \cdot 8 \cdot 7 = 10000 - 5040 = 4960$.

Ex: How many old and new Quebec licence plates?

$10^3\, 26^3 + 26^4\, 10^2 = 17.576000 + 45.697600 = 63\,273600$.

Ex: How many IPv4 and IPv6, Internet Protocol addresses.

IPv4 $\rightarrow$ 16 bits for netid (identifies the network) and 16 bit for hostid (identifies the device) $\rightarrow 2^{16} \times 2^{16} = 2^{32} \approx 4.3 \times 10^9$

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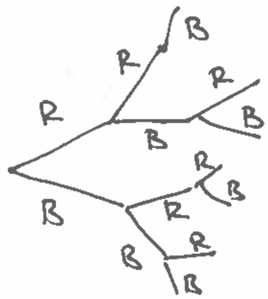
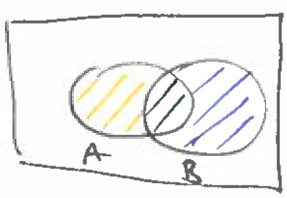
IPv6 → 64 bit netid and 64 bit host id → $2^{64} \times 2^{64} = 2^{128} =$
$= 3.4 \times 10^{38}$

Decision Trees:

Ex: An urn contains 2 red and 3 blue balls. 3 balls are removed without replacement. What are the possible outcomes.



Constr:



Notice that $A \setminus B$, $B \setminus A$, $A \cap B$ are disjoint.

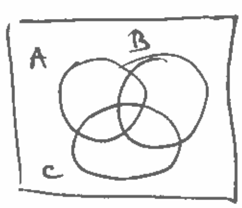
Thus: $|A \cup B| = |A \setminus B| + |A \cap B| + |B \setminus A|$

$\Rightarrow |A \cup B| = |A| + |B| - |A \cap B|$.

Ex: Robotics club → 35 CS majors, 26 Math majors with a total of 50 members. How many double majors

$50 = 35 + 26 - x \qquad x = 11.$

Constr:



$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

Ex: 207 employees; 114 have life insurance, 152 health insurance 25 dental; 64 both life and health, 12 both life and dental and 21 both health and dental and 8 all three how many have no insurance whatsoever?

$(L \cup H \cup D) = 114 + 152 + 25 - 64 - 12 - 21 + 8 = 202$

$207 - 202 = 5$ with no insurance of any sort.

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Principle of Inclusion and Exclusion: Given finite sets $A_1, \dots, A_n$ the

$$|A_1 \cup \dots \cup A_n| = \sum_{1 \le i \le n} |A_i| - \sum_{1 \le i < j \le n} |A_i \cap A_j| + \sum_{1 \le i < j < k \le n} |A_i \cap A_j \cap A_k| - \dots + (-1)^{n+1} |A_1 \cap \dots \cap A_n|.$$

Pr: By Induction in the book. $\boxtimes$

Principle: Pigeonhole Principle. If more than k items are placed in k bins, then at least 1 bin contains more than 1 item.

Ex: 20 students in the room. At least 2 student are born in the same month.

Ex: Prove that if 51 integers between 1 and 100 are chosen, then one of them must divide another.

Pr: Let the integers be $n_1, \dots, n_{51}$; $\forall n_i \ge 2$ has a prime factorizatio[n] $n_i = 2^{k_i} b_i$, b_i - odd. There are 50 odd integers between 1 and 9[9] inclusive, but we have 51 b-values. By the Pigeonhole Principl[e] $b_i = b_j$ for some i, j and then $n_i = 2^{k_i} b_i$, $n_j = 2^{k_j} b_i$ so that $n_i | n_j$ or $n_j | n_i$. $\boxtimes$

§4.2 p 259

16, 32, 40, 48, 58, 66

§4.3 p 270

8, 14, 26