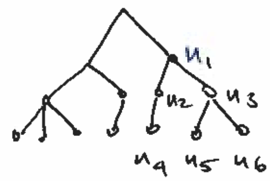
Disc Math - §19 - Trees

Def: A tree is an acyclic, connected graph. When one node is designated as the root we have a rooted tree. Forest - acyclic grap (collection of trees).

ex:



u_2, u_3 are the children of u_1 (u_1 is the parent of u_2, u_3).

def: In a rooted tree:

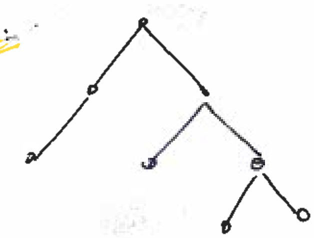
- the depth of a node is the length of the path from the root to that node (the root has depth 0).
- The depth (height) of the tree is the max depth of any node
- a node with no children is a leaf of the tree

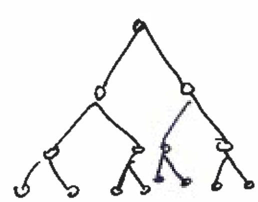
ex: u_4, u_5, u_6 are leaves of the tree above.

Def: A binary tree is rooted tree where every node has at most two children. In a binary tree each child is designated as a left child or a right child.

A full binary tree is a binary tree where all internal nodes have two children and all leaves are of the same depth.

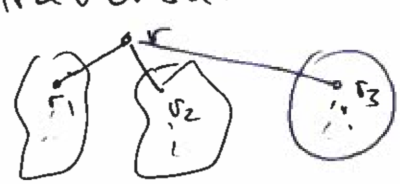
ex:





ex: Files and Folders on Computer are organized as a tree. Is it binary. What are the leaves.

Tree traversal alorithms: preorder, inorder, postorder.



Mention, but do not explain.

Disc Math - § - Trees

Prop: A tree with n nodes has $n-1$ arcs.

Pr: By induction. Base case $n=1$, one node, no arcs. Assume trees with k nodes have $k-1$ arcs.

Let a tree have $k+1$ nodes. Let x be a leaf. Remove x and the adjoining arc. The remainder is a tree with k nodes and $k-1$ arcs. Our tree has $k+1$ nodes and k arcs. ☒

Prop: A full binary tree of depth k has 2^k leaves.

Pr: Obvious.

Cor: A full binary tree of depth k has 2^k-1 internal nodes (including the root).

Pr: $1 + 2^1 + 2^2 + \dots + 2^{k-1} = \sum_{i=0}^{k-1} 2^i = \frac{2^k-1}{2-1} = 2^k - 1$. ☒

HW §6.2 p 521

4, 39, 41, 47, 52