Probstat - Clex 1 - Solutions

(1) $P(A\cap B) = P(A) + P(B) - P(A\cup B) \geq P(A) + P(B) - 1 = \frac{3}{4} + \frac{1}{3} - 1 = \frac{1}{12}$

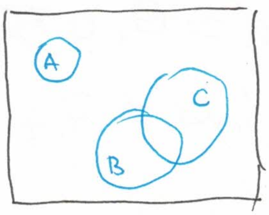
$P(A\cap B) \leq \min\{P(A), P(B)\} = \min\{\frac{3}{4}, \frac{1}{3}\} = \frac{1}{3}$

$P(A\cup B) \leq \min\{P(A)+P(B), 1\} = \min\{\frac{13}{12}, 1\} = 1$

$P(A\cup B) \geq \max\{P(A), P(B)\} = \max\{\frac{3}{4}, \frac{1}{3}\} = \frac{3}{4}$

$3/4 \leq P(A\cup B) \leq 1.$

(2)



a) $P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(B\cap C) = 0.2+0.4+0.6-0.3 = 0.9$

b) $P(A'\cap B\cap C) = P(B\cap C) = 0.3$

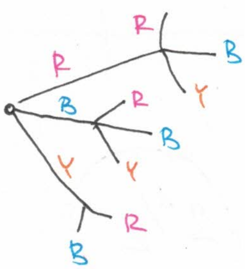
c) $P(A'\cap B'\cap C') = 1 - P(A'\cap B'\cap C')' = 1 - P(A\cup B\cup C) = 1 - 0.9 = 0.1$

d) $P((A\cup B)\cap C) = P((A\cap C)\cup(B\cap C)) = P(\emptyset\cup(B\cap C)) = P(B\cap C) = 0.3$

e) $P((A\cup B')\cap C') = P(B'\cap C') = 1 - P(B'\cap C')' = 1 - P(B\cup C) =$

$= 1 - [P(B) + P(C) - P(B\cap C)] = 1 - [0.4 + 0.6 - 0.3] = 0.3$

(3)



a) 8

b) 5

c) 3

(4) a) $A\cap B = \{b, f\}$, b) $A\cup B = \{a, b, c, f\}$, c) $A' = \{c, d, e\}$

d) $(A\cup C)' = \{d\}$, e) $A'\cap C' = \{d\}$

(5) a) $(A\cup B)\cap C = (2, 4)$, b) $(A\cup C)\cap B = [3, 4)$

c) $(A\cup B)\cap(A\cup C) = [1, 4)$

(6) a) $|A'\cap B| = 10$, b) $|B'| = 2+8 = 10$, c) $|A\cup B| = 80+10+2 = 92$

(7) $A' = \{x \mid x \geq 72.5\}$ - rise time is at least 72.5 min

$B' = \{x \mid x \leq 52.5\}$ - rise time is at most 52.5 min

$A\cap B = \{x \mid 52.5 < x < 72.5\}$ - rise time more than 52.5 min, but less than 72.5 min

Probstat - Clex 1 - Solutions

$A \cup B = \{x \mid x > 0\}$ - rise time is positive.

(8) a) $p(1 \text{ nonzero}) = 4/16 = 1/4$

b) $p(\text{invertible}) = 6/16 = 3/8$

(9) S - total number of oranges in a crate

B - orange has a bruise on it

W - orange has a worm

$$|(B \cup W)'| = |S| - |B \cup W| = |S| - (|B| + |W| - |B \cap W|) =$$

$$= 150 - [15 + 20 - 10] = 75$$

(10) C_1 - divisible by 2^2; C_2 - divisible by 3^2; C_3 - divisible by 5^2

C_4 - divisible by 7^2; C_5 - divisible by 11^2

$$|C_1| = \left\lfloor \frac{150}{2^2} \right\rfloor = 37, \quad |C_2| = \left\lfloor \frac{150}{3^2} \right\rfloor = 16, \quad |C_3| = \left\lfloor \frac{150}{5^2} \right\rfloor = 6, \quad |C_4| = \left\lfloor \frac{150}{7^2} \right\rfloor = 3$$

$$|C_5| = \left\lfloor \frac{150}{11^2} \right\rfloor = 1, \quad |C_1 \cap C_2| = \left\lfloor \frac{150}{2^2 \cdot 3^2} \right\rfloor = 4, \quad |C_1 \cap C_3| = \left\lfloor \frac{150}{2^2 \cdot 5^2} \right\rfloor = 1$$

$$p((C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5)') = 1 - p(C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5) =$$

$$= 1 - (p(C_1) + p(C_2) + p(C_3) + p(C_4) + p(C_5) - p(C_1 \cap C_2) - p(C_1 \cap C_3)) =$$

$$= 1 - \frac{1}{150}(37 + 16 + 6 + 3 + 1 - 4 - 1) = 1 - \frac{58}{150} = \frac{46}{75} = 0.6133$$