PROBABILITY AND STATISTICS, CLASS EXERCISE 1

(1) Let A and B be events with probabilities $P(A) = 3/4$ and $P(B) = 1/3$. Show that $1/12 \leq P(A \cap B) \leq 1/3$. Find the corresponding bounds for $P(A \cup B)$.

(2) Let A, B and C be three events such that A is disjoint from both B and C, $P(A) = 0.2, P(B) = 0.4, P(C) = 0.6$ and $P(B \cap C) = 0.3$. Determine the following probabilities:

a) $P(A \cup B \cup C)$,
b) $P(A' \cap B \cap C)$,
c) $P(A' \cap B' \cap C')$,
d) $P((A \cup B) \cap C)$,
e) $P((A \cup B') \cap C')$.

(3) There are six balls in an urn. They are identical except for color. Two are red, three are blue, and one is yellow. You are to draw one ball from the urn, note its color, and set it aside, then draw another ball and note its color.

(a) How many sample points are in the sample space?
(b) How many sample points have at least one red ball in the sequence?
(c) How many sample points have no red in the sequence?

(4) Consider the sample space $S = \{a, b, c, d, e, f\}$. Consider the events $A = \{a, b, f\}, B = \{b, c, f\}$, and $C = \{c, e, f\}$. Compute

(a) $A \cap B$
(b) $A \cup B$
(c) A'
(d) $(A \cup C)'$
(e) $A' \cap C'$

(5) Consider the sample space, $S = \mathbb{R}^+$. Let $A = [1, 3), B = [3, 5]$, and $C = (2, 4)$. Compute

(a) $(A \cup B) \cap C$
(b) $(A \cup C) \cap B$

(c) $(A \cup B) \cap (A \cup C)$

(6) Samples of a cast Aluminum part are classified on the basis of surface finish (in microinches) and edge finish. The results of 100 parts are summarized below:

	Excellent Edge Fin.	Good Edge Fin.
Excellent Surface Fin.	80	2
Good Surface Fin.	10	8

Let A denote the event that the sample has excellent surface finish, and let B denote the event that the sample has excellent edge finish. Determine the number of samples in

(a) $A' \cap B$
(b) B'
(c) $A \cup B$

(7) The rise time of a reactor is measured in minutes (and fractions of minutes). Let the sample space be the positive, real numbers. Define the events A and B as follows

$$A = \{x \,|\, x < 72.5\}$$
$$B = \{x \,|\, x > 52.5\}$$

Describe the following events

(a) A'
(b) B'
(c) $A \cap B$
(d) $A \cup B$

(8) Let S be the sample space of all 2×2 matrices for which the entries are either 0 or 1.

$$S = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

(a) What is the probability that a randomly selected matrix contains exactly one nonzero entry in it?
(b) What is the probability that a randomly selected matrix in S is invertible?

(9) In recent years, criminals have used plaster casts, breast implants, and hollowed out pineapples to smuggle cocaine in and out of countries. Last January, two cars, stopped by Spanish police were found to be completely full of oranges[1]. Four tonnes were seized! The five people responsible boldly claimed that they were legitimately purchased for personal consumption - which suggests that they were planning to consume a couple of thousand of oranges each.

In a crate of 100 oranges; there are 20 that have worms in them and 15 that have bruises. Only oranges with neither worms nor bruises can be sold. If there are 10 bruised oranges that have worms in them, how many of the 100 oranges can be sold?

(10) Pick a number at random in the interval $[1, 150]$. What is the probability that the number is square free? An integer, n, is called square free if it does not have a divisor of the form k^2 where $k \in \{2, 3, \ldots, n\}$.

[1]https://www.businessinsider.com/police-pull-over-family-with-tons-of-oranges-in-car-2018-2