

Probstats - Clex 11 - solutions

① a) $\alpha = 0.02$, $z_{\alpha/2} = 2.33$; $4.56 \pm 2.33 \frac{0.75}{\sqrt{16}} \Rightarrow$
 $4.12 \leq \mu \leq 5.00$ with 98% confidence

b) $n = \left(\frac{z_{\alpha/2} \sigma}{E} \right)^2 = \left(\frac{(2.33)(0.75)}{0.2} \right)^2 = 76.3 \Rightarrow n = 77$

② $\bar{x} = 64.25$, $s^2 = 6.5$, $s = 2.55$

$64.25 \pm 2.365 \frac{2.55}{\sqrt{8}} \Rightarrow 62.13 \leq \mu \leq 66.37$ with 95% confidence

Cannot claim the height has changed since the old height of 63 in is in the interval.

③ $H_0: \mu = 70$, $H_1: \mu > 70$; $z = \frac{71.8 - 70}{8.9/\sqrt{100}} = 2.02$

p-value = $p(z > 2.02) = 1 - 0.9783 = 0.0217$

a) $\alpha = 0.05 > p\text{-value} = 0.0217$. Reject H_0 . Accept H_1 .

b) $\alpha = 0.01 < p\text{-value} = 0.0217$. Fail to reject H_0 .

c) p-value = 0.0217. To get a more definite answer study a larger sample.

④ $\bar{x} = 64.35$, $s = 2.55$; $H_0: \mu = 63$, $H_1: \mu > 63$

$t = \frac{64.25 - 63}{2.55/\sqrt{8}} = 1.386$ with 7 d.f., p-value = 0.1041 $> \alpha = 0.05$

Fail to reject H_0 . This sample does not provide enough evidence to claim that the average height has increased.

⑤ $\mu \leq \bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}} = 6.2 + 1.645 \frac{2}{\sqrt{25}} = 6.858$ sec with 95% confidence

⑥ $\bar{x} = 5.233$, $s^2 = 0.02267$, $s = 0.15055$

$5.233 \pm 3.365 \frac{0.15055}{\sqrt{6}} \Rightarrow 5.026 \leq \mu \leq 5.44$ with 98% confidence

⑦ $H_0: \mu = 250$, $H_1: \mu < 250$; $z = \frac{240 - 250}{140/\sqrt{110}} = -3.60$

p-value = $p(z < -3.60) = 0.0013 < \alpha = 0.01$. Reject H_0 . Accept H_1 .

8) a) $H_0: \mu = 40, H_1: \mu < 40; z = \frac{37 - 40}{8/\sqrt{91}} = -3.375$

p-value = $P(z < -3.375) = 0.000369 < \alpha = 0.05$ Reject H_0 . Accept H_1 .

The parents are not correct. It takes less than 40 min to prepare dinner.

b) $H_0: \mu = 30, H_1: \mu < 30; z = \frac{28.5 - 30}{10/\sqrt{81}} = -1.35$

p-value = $P(z < -1.35) = 0.088508 > \alpha = 0.05$ Fail to reject H_0 .

There is not enough evidence in this sample to claim that the clean-up time is less than 30 min. The sample does not support the parents claim.

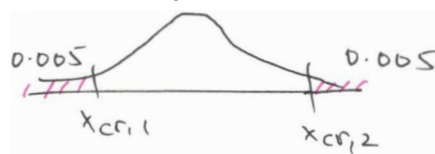
9) a) $\alpha = 0.01, z_{\alpha/2} = 2.575$

$\chi_{cr,1} = 30000 - 2.575 \frac{1500}{\sqrt{16}} = 29032.5$

$\chi_{cr,2} = 30000 + 2.575 \frac{1500}{\sqrt{16}} = 30967.5$

$z_1 = \frac{29032.5 - 31000}{1500/\sqrt{6}} = -5.25; z_2 = \frac{30967.5 - 31000}{1500/\sqrt{6}} = -0.09$

$\beta = P(-5.25 < z < -0.09) = 0.4641; \text{Power} = 1 - \beta = 0.5359$



10) $H_0: \mu = 160, H_1: \mu > 160$

$t = \frac{180 - 160}{36/\sqrt{27}} = 2.887$ with 26 d.f. p-value = $0.0039 < \alpha = 0.05$

Reject H_0 . Accept H_1 . There is strong evidence that the average farm size has increased.

11) $H_0: \mu = 200, H_1: \mu > 200; \bar{x} = 249.6667, s = 145.149$

$t = \frac{249.6667 - 200}{145.149/\sqrt{12}} = 1.185$ with 11 d.f.; p-value = $0.1305 > \alpha$

Fail to reject H_0 . This sample does not support sufficiently the claim that the average repair time exceeds 200 min.

Probstat Cle x 10 - Solutions

(12) a) $25 \pm 1.645 \frac{1.25}{\sqrt{9}} \Rightarrow 24.31 \leq \mu \leq 25.69$ with 90% confidence

b) $25 - 2.054 \frac{1.25}{\sqrt{9}} \Rightarrow \mu \geq 24.14$ with 98% confidence.

(13) a) $32 \pm 2.7238 \frac{5}{\sqrt{36}} \Rightarrow$

$29.73 \leq \mu \leq 34.27$ with 99% confidence.

b) The distance of $X=74$ to the upper bound of the above CI is

$t = \frac{74 - 34.27}{5} = 7.946$. This case is very extreme right

tail in the t -distribution. The probability for observing dissolving time this long or longer is

$P(t > 7.946) \approx 1.2 \times 10^{-9}$

The case against Pepsi Cola is very weak (inexistent).

(14) $H_0: \mu = 10$, $H_1: \mu < 10$

$t = \frac{8.5 - 10}{1.75/\sqrt{14}} = -3.207$ with 13 d.f.

$P\text{-value} = 0.0034 < \alpha < 0.01$

Reject H_0 . Accept H_1 . The amount of weight loss on Master Cleanse is less than 10 pounds in 10 days.