

PROBABILITY AND STATISTICS, CLASS EXERCISE 10

- (1) Assume that the helium porosity (in percentage) of coal samples taken from a particular seam is normally distributed with true standard deviation of 0.75.
- Compute a 98% confidence interval for the true average porosity based on data from 16 specimens with sample average porosity of 4.56.
 - What sample size is needed to estimate the true average porosity within 0.2 with 98% confidence?
- (2) A researcher believes that in recent years women have been getting taller. She knows that 10 years ago the average height of young adult women living in her city was 63 inches. The standard deviation is unknown. She randomly samples eight young women residing in her city and measures their heights. The following data is obtained:

64 66 68 60 62 65 66 63

Based on this sample, can the reasearcher state with 95% confidence that the average height of women changed in the past 10 years? Assume the population is normal.

- (3) A random sample of 100 recorded deaths in the US during the past year showed an average life span of 71.8 years. Assuming a normal population and a population standard deviation of 8.9 years, does this seems to indicate that the mean life span today is greater than 70 years?
- Use 0.05 level of significance.
 - Use 0.01 level of significance.
 - What is the p -value of this test. What is your final conclusion?
- (4) A researcher believes that in recent years women have been getting taller. She knows that 10 years ago the average height of young adult women living in her city was 63 inches. The standard deviation is unknown. She randomly samples eight young women residing in her city and measures their heights. The following data is obtained:

64 66 68 60 62 65 66 63

Based on this sample, can the reasearcher state that the average height of women changed in the past 10 years? Implement a statistical test at the

$\alpha = 0.05$ level of significance. Assume the population is normal.

- (5) In a psychological testing experiment, 25 subjects are selected randomly and their reaction time, in seconds, to a particular stimulus is measured. Past experiments have shown that the standard deviation of reaction time to this kind of stimuli is 2 seconds and that reaction times are approximately normally distributed. The average time for the 25 subjects was 6.2 seconds. Give an upper 95% confidence bound for the mean reaction time.
- (6) A small brewery monitors the alcohol content of its beer by taking 6 samples from every vat brewed. The following are the percent alcohol content data for the sample taken from the last vat:

5.3 5.0 5.1 5.4 5.3 5.3

- a) Calculate the mean and the standard deviation for the alcohol content of the sample.
- b) Assuming the population is approximately normally distributed construct a 98% confidence interval for the mean alcohol content.
- (7) A random sample of size $n = 110$ is drawn from a population. The sample mean is 210 and the sample standard deviation is 140. Test the null hypothesis that the population mean is 250 versus the alternative that it is less than 250. Use a 1% level of significance.
- (8) Amelia's parents claim that it takes them on average 40 minutes to put a meal on the table for dinner and therefore Amelia should wash the dishes and clean the kitchen after dinner, which according to the parents takes less than 30 minutes. Amelia collected data for 81 days which shows mean dinner preparation time of 37 minutes with standard deviation of 8 minutes and mean clean-up time of 28.5 minutes with standard deviation of 10 minutes.
- a) For the dinner preparation time test $H_0 : \mu = 40$ against $H_1 : \mu < 40$ at 5% level of significance. Are the parents correct in their claim it takes 40 minutes to prepare dinner?
- b) For the clean-up time test $H_0 : \mu = 30$ against $H_1 : \mu < 30$ at 5% level of significance. Are the parents correct in their claim it takes less than 30 minutes to clean up after dinner?
- (9) Let μ denote the true average tread life of a certain type of tire. Consider testing $H_0 : \mu = 30000$ versus $H_1 : \mu \neq 30000$ based on a sample of size $n = 16$ from a normal population distribution with $\sigma = 1500$ at $\alpha = 0.01$

level of significance. What is the probability of making a type II error when $\mu = 31000$? What is the power of the test?

- (10) Five years ago, the average size of farms in a western province was 160 acre, From a recent survey of 27 farms, the sample mean and the standard deviation were found to be 180 and 36 acres, respectively. Is there a strong evidence that the average farm size is larger today then it was 5 years ago? Assuming that the population is approximately normal, formulate and implement a statistical test.
- (11) The article "Uncertainty Estimation in Railway Track Life-Cycle Cost" (J. of Rail and Rapid Transit, 2009) presented the following data on time to repair (in min) a rail break in the high rail on a curved track of a certain railway line:

159 120 480 149 270 547 340 43 228 202 240 218

The population distribution is approximately normal. Is there a compelling evidence for concluding that the true average repair time exceeds 200 min? Formulate and implement a statistical test at $\alpha = 0.05$ level of significance.

- (12) Suppose that the weight of vintage Ken doll is normally distributed with a known standard deviation of 1.25 grams. A sample of nine Ken dolls has a sample mean of 25 grams.
- Construct and interpret a two-sided 90% confidence interval for the true weight of all vintage Ken dolls.
 - Construct and interpret a one-sided 98% lower confidence bound for the average weight of a vintage Ken dolls.
- (13) In 2009, a man tried to sue PepsiCo after he allegedly found a mouse in his can of Mountain Dew. Lawyers for the soft drink giant, refuted the man's claim by stating that Mountain Dew *could* dissolve a mouse in 30 days, and showed that his can was purchased 74 days after the container had been sealed. The case was settled out of court.
- Scientists for PepsiCo, tested 36 samples of Mountain Dew and found that it took an average of 32 days with a standard deviation of 5 days for the beverage to dissolve a mouse completely. Assume that the time its takes for soda to dissolve a mouse is normally distributed.
- Construct a two-sided 99% confidence interval for the actual number of days that it takes for the soda to dissolve a mouse and interpret the interval in the context of the problem.

b) If a person claims to have found a mouse in their can of Mountain Dew, 74 days after the soda was dispensed into the can, would they have a solid case against PepsiCola? Why or why not?

- (14) A popular weight loss method among celebrities these days is the “Master Cleanse”. The detox regime involves avoiding food for about 10 days, drinking a lemonade mixture flavoured with maple syrup and cayenne pepper, and consuming a gallon of saltwater on top of that. The only break from the monotony of this liquid diet is a delicious laxative before bed. Singer Beyoncé and actress Gwyneth Paltrow swear by it.

“Master Cleanse” claims that you can lose an average of 10 pounds in 10 days on this diet. However, in a random sample of 14 dieters on this regime, it was found that their average weight loss in 10 days was only 8.5 pounds, with a standard deviation 1.75 pounds. Assume that the dieters were selected from a normally distributed population. At the 1% level of significance, does the data indicate that the amount of weight lost on Master Cleanse is less than 10 pounds in 10 days?