ProbStat - Clex 11 - Solutions

(1) $s_1^2 = 59.028$, $s_2^2 = 15.278$

$$f = \frac{59.028}{15.278} = 3.864 \text{ with } (8,8) \text{ d.f.}$$

p-value $= 0.0367 < 0.05 = \alpha$

Reject H_0. Accept H_1. There is more variability in the test scores of unmarried women.

(2) $$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{11(0.771)^2 + 9(0.448)^2}{12+10-2} = 0.417$$

$s_p = \sqrt{0.417} = 0.646$

$$(3.11 - 2.04) \pm 1.725(0.646)\sqrt{\frac{1}{12}+\frac{1}{10}}$$

$0.593 < \mu_1 - \mu_2 < 1.547$ with 90% confidence

(3) $H_0: \mu_1 - \mu_2 = 0$, $H_1: \mu_1 - \mu_2 < 0$

$n_1 = 10$, $\bar{x}_1 = 2902.8$, $s_1 = 277.3$; $n_2 = 8$, $\bar{x}_2 = 3108.1$, $s_2 = 205.9$

$$\nu = \frac{\left(\frac{277.3^2}{10} + \frac{205.9^2}{8}\right)^2}{\left(\frac{277.3^2}{10}\right)^2/9 + \left(\frac{205.9^2}{8}\right)^2/7} = 15.94 \rightarrow 15 \text{ d.f.}$$

$$t = \frac{2902.8 - 3108.1}{\sqrt{\frac{(277.3)^2}{10} + \frac{(205.9)^2}{8}}} = -1.8 \text{ ; p-value} = 0.046 < 0.05 = \alpha$$

Reject H_0. Accept H_1. Fusion increases tensile strength.

(4) $$z = \frac{(\bar{X}_A - \bar{X}_B) - (\mu_A - \mu_B)}{\sqrt{\sigma_A^2/n_A + \sigma_B^2/n_B}} = \frac{1.5 - 0.5}{\sqrt{(1.2)^2/18 + (1.2)^2/18}} = 2.5$$

$$P(\bar{X}_A - \bar{X}_B > 1.5) = P(Z > 2.5) = 0.00621$$

5 a) $$\frac{s_1^2}{s_2^2} f_{0.975}(8,10) \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} f_{0.025}(8,10)$$

$$\frac{16}{25} \cdot 0.232822 \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{16}{25} \cdot 3.854891$$

$$0.149 \le \sigma_1^2/\sigma_2^2 \le 2.467$$ with 95% confidence.

Since 1 is in the interval we cannot claim that the variances differ

b) Pooled estimator.

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{(10)(16) + (8)(25)}{11+9-2} = 20$$

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} s_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = (85-81) \pm 2.1009\sqrt{20}\sqrt{\frac{1}{11}+\frac{1}{9}}$$

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} s_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 4 \pm 4.2230$$

$$-0.223 \le \mu_1 - \mu_2 \le 8.223$$ with 95% confidence.

Since 0 is in the interval we cannot claim that the outputs of the two processes differ with 95% confidence.