

PROBABILITY AND STATISTICS, CLASS EXERCISE 11

- (1) The following are the scores obtained in a personality test for samples of nine married women and nine unmarried women:

Unmarried	88	68	77	82	63	80	78	71	72
Married.	73	77	67	74	74	64	71	71	72

Assuming that these data can be looked upon as independent random samples from two normal populations, test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against the one-sided alternative $\sigma_1^2 > \sigma_2^2$ at $\alpha = 0.05$ level of significance. Make sure to estimate the p -value of this test and draw a conclusion in the context of the problem.

- (2) Two independent sampling stations are chosen for a study, one located downstream from an acid mine discharge point and the other located upstream. For 12 samples collected at the downstream station the species diversity index has mean value $\bar{x}_1 = 3.11$ and st. dev. $s_1 = 0.771$, while 10 samples collected at the upstream station had $\bar{x}_2 = 2.04$ and $s_2 = 0.448$. Determine a 90% confidence interval for the difference of population means assuming that the populations are normally distributed with equal variances.
- (3) The deterioration of many municipal pipeline networks across the country is a growing concern. One technology proposed for pipeline rehabilitation uses a flexible liner threaded through existing pipe. The article "Effects of welding on a high-density polyethylene liner" (*J. Material. Civil Eng.*) reported the following data on the tensile strength (psi) of liner specimens both when a certain fusion process was used and when this process was not used.

No fusion	2748	2700	2655	2822	2511	3149	3257	3213	3220	2753
Fused	3027	3356	3359	3297	3125	2910	2889	2902		

Assuming that the two populations are normally distributed test the hypothesis that fusion does not affect the tensile strength at the $\alpha = 0.05$ level of significance.

- (4) Two independent experiments are being run in which two different types of paints are compared. Eighteen specimens are painted using type A and the drying time, in hours is recorded for each. The same is done with type B paint. The two populations are approximately normal and the standard deviations are both known to be 1.2 hours. Assuming that the true mean drying time for paint A is half an hour longer than the mean drying time for paint B, find $p(\bar{X}_A - \bar{X}_B > 1.5)$ for samples of size $n_A = n_B = 18$.
- (5) In a batch chemical process, two catalysts are being compared for their effect on the output of the process reaction. A sample of 11 batches was prepared using catalyst 1 and a sample of 9 batches was prepared using catalyst 2. The sample results are as follows:

$$n_1 = 11, \quad \bar{x}_1 = 85, \quad s_1^2 = 16; \quad n_2 = 9, \quad \bar{x}_2 = 81, \quad s_2^2 = 25$$

Assume that the output is normally distributed for both processes.

a) Construct a 95% confidence interval for the ratio of variances, σ_1^2/σ_2^2 . Based on this confidence interval can you claim that the variances differ? Use this conclusion to establish if you can use a pooled estimator for the variances in b).

b) Construct a 95% confidence interval for the difference of means, $\mu_1 - \mu_2$. Based on this confidence interval can you claim that the two processes have different output?