PROBABILITY AND STATISTICS, CLASS EXERCISE 12

(1) An instructor has given a short quiz consisting of two parts. For a randomly selected student, let X be the number of points earned on the first part and Y be the number of points earned on the second part. Suppose that the joint pmf of X and Y is given by the accompanying table:

		y			
	$p(x,y)$	0	5	10	15
	0	0.02	0.06	0.02	0.10
x	5	0.04	0.15	0.20	0.10
	10	0.01	0.15	0.14	0.01

a) Compute the marginal probability distributions of X and Y.

b) Compute the conditional probability mass function of Y given that $X = 10$.

c) Compute the conditional mean and variance of X given that $Y = 5$.

d) Are X and Y independent; justify your conclusion numerically.

(2) Compute the covariance and the correlation between the RV's X and Y whose joint pmf is given in the following table

		y		
	$p(x,y)$	0	1	2
	0	0.12	0.05	0.03
x	2	0.05	0.35	0.10
	5	0.03	0.10	0.17

(3) In a certain town, 40% of the eligible voters prefer candidate A, 10% prefer candidate B, and the remaining 50% have no preference. You randomly sample 12 eligible voters.

a) What is the probability that 6 will prefer candidate A, 2 will prefer candidate B, and the remaining 4 will have no preference?

b) What is the marginal distribution for the number of voters who prefer candidate A in your sample?

c) What is the marginal joint pmf for the number of voters who prefer candidates A, B in your sample?

d) What is the conditional joint distribution for the number of voters who prefer candidates A, B in your sample given that 5 voters in the sample have no preference?

(4) Two random variables X and Y have joint probability density function

$$p(x, y) = 24xy, \qquad 0 < x < 1, \;\; 0 < y < 1 - x$$

a) Determine the marginal densities.

b) Determine the conditional density for X conditioned on $Y = y$. Determine the conditional density for Y conditioned on $X = x$.

c) Determine the conditional mean and the conditional variance of X conditioned on $Y = 1/4$.

d) Are the variables X and Y independent? Justify your conclusion.

(5) Let $X_1, X_2, \ldots, X_k$ be the counts in a multinomial experiment with n trials and k classes and with corresponding probabilities $p_1, p_2, \ldots, p_k$. Show that the correlation between X_i and X_j is given by

$$corr(X_i, X_j) = \frac{-p_i p_j}{\sqrt{p_i(1 - p_i)p_j(1 - p_j)}}$$