Probstat - Clex 2 - Solutions

1) a) $P(A|B)P(B) = P(A\cap B) = P(B|A)P(A) \Rightarrow P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$

b) $\dfrac{P(B|A)}{P(B)} = \dfrac{P(A|B)}{P(A)} > 1$

2) a) $p(S) = p((A\cap B)\cup(C\cap D\cap E)) = P(A\cap B) + P(C\cap D\cap E) - P(A\cap B\cap C\cap D\cap E) =$

$= (0.7)(0.8) + (0.8)(0.7)(0.9) - (0.7)(0.8)(0.8)(0.7)(0.9) = 0.7818$

b) $p(A'|S) = \dfrac{1}{p(S)}\, p(A'\cap S) = \dfrac{1}{p(S)}\, p[A'\cap((A\cap B)\cup(C\cap D\cap E))] =$

$= \dfrac{1}{p(S)}\, p(A'\cap C\cap D\cap E) = \dfrac{1}{0.7818}(0.3)(0.8)(0.7)(0.9) = 0.1934$

3) $p(B|A) = \dfrac{p(B\cap A)}{p(A)} = \dfrac{40+16}{12+40+44+16} = \dfrac{56}{112} = \dfrac{1}{2}$

$p(B) = \dfrac{92}{264} = 0.451 \neq 0.5 = p(B|A) \Rightarrow$ A & B are not independent

4) The composition of the urn is not deterministically fixed before the final draw. It is true that the collection (two black, one white) gives probability of drawing a black ball at random. However, many other probabilistic states of the urn give the same probability. Amongst these probabilistic states is $p(BBB) = p(BWW) = 1/4$, $p(BBW) = 1/2$, which also gives a probability of drawing a black ball equal to $2/3$. Carroll effectively confounds state of knowledge with state of Nature.

5) a) $p(F_1'\cap F_2') = p(F_1')\,p(F_2') = (0.04)^2 = 0.0016$

b) $p(F_1\cup F_2) = 1 - p(F_1'\cap F_2') = 1 - 0.0016 = 0.9984$

6) $P((A\cup B)\cap C) = P((A\cap C)\cup(B\cap C)) = p(A\cap C) + p(B\cap C) - p(A\cap B\cap C) =$

$= p(A)p(C) + p(B)p(C) - p(A)p(B)p(C) = [p(A) + p(B) - p(A\cap B)]\,p(C)$

$= P(A\cup B)\,P(C)$

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7) $P(D) = 0.42$, $P(C) = 0.55$, $P(C|D) = 0.35$

a) $P(D|C) = \dfrac{P(C|D)P(D)}{P(C)} = \dfrac{(0.35)(0.42)}{0.55} = 0.2673$

b) $P(D' \cap C') = 1 - P(D \cup C) = 1 - [P(D) + P(C) - P(D \cap C)] =$

$= 1 - [0.42 + 0.53 - (0.35)(0.42)] = 0.177$

8) $P(S|F) = \dfrac{P(F|S)P(S)}{P(F|S)P(S) + P(F|M)P(M) + P(F|L)P(L)} = \dfrac{(0.10)(0.45)}{(0.10)(0.45) + (0.12)(0.35) + (0.15)(0.20}$

$P(S|F) = 0.385$. Similarly, $P(M|F) = 0.389$, $P(L|F) = 0.256$

9) a) $P(K|D) = \dfrac{P(D|K)P(K)}{P(D|K)P(K) + P(D|J)P(J)} = \dfrac{(0.02)(0.4)}{(0.02)(0.4) + (0.03)(0.6)} = \dfrac{0.008}{0.026} = 0.307$

b) $P(J|D') = \dfrac{P(J \cap D')}{P(D')} = \dfrac{P(D'|J)P(J)}{1 - P(D)} = \dfrac{(1-0.03)(0.6)}{1 - 0.026} = 0.5975$

c) $P(J \cup D') = P(J) + P(D') - P(J \cap D') = P(J) + P(D') - P(D'|J)P(J) =$

$= (0.60) + (1 - 0.026) - (0.97)(0.60) = 0.992$

10) $P(A') = 10^{-5}$; $P(B'|A') = 10^{-1}$; $P(B'|A) = 10^{-5}$; $P(C'|A' \cap B') = 10^{-1}$

All 3 parts failed:

$P(A' \cap B' \cap C') = P(C'|A' \cap B')P(A' \cap B') = P(C'|A' \cap B')P(B'|A')P(A') =$

$= 10^{-1} \cdot 10^{-1} \cdot 10^{-5} = 10^{-7}$

Two of 3 parts failed: (The Three events are disjoint)

$P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C) =$

$= P(C'|A \cap B')P(B'|A)P(A) + P(C'|A' \cap B)P(B|A')P(A') + P(C|A' \cap B')P(B'|A')P(A$

$= 10^{-2} \cdot 10^{-5}(1 - 10^{-5}) + 10^{-2}(1 - 10^{-1}) \cdot 10^{-5} + (1 - 10^{-1}) \cdot 10^{-1} \cdot 10^{-5}$

$= 10^{-6} + 10^{-7} - 10^{-8} - 10^{-12}$

P(3 Parts failed | At leas 2 parts failed) $= \dfrac{10^{-7}}{(10^{-6} + 10^{-7} - 10^{-8} - 10^{-12}) + 10^{-7}} = 0.08$

The probability That I will make Things worse is greater Than 1%, so I shouldn't try to fix it.

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11) $P(C)=0.7$, $P(L)=0.15$, $p(L|C')=0.4$

b) $$P(C'|L)=\frac{p(C'\cap L)}{p(L)}=\frac{p(L|C')p(C')}{p(L)}=\frac{(0.4)(1-0.7)}{0.15}=0.8$$

a) $p(C|L)=1-p(C'|L)=1-0.8=0.2.$

12) a) $p(C_2|C_1)=0.20$, $P(C_1|C_2)=0.33$, $P(C_1\cup C_2)=1.$

$$P(C_2|C_1)P(C_1)=P(C_1\cap C_2)=P(C_1|C_2)p(C_2)$$

$$0.2\,P(C_1)=P(C_1\cap C_2)=0.33\,P(C_2)\ \Rightarrow\ P(C_1)=1.65\,P(C_2)$$

$$P(C_1\cup C_2)=P(C_1)+P(C_2)-P(C_1\cap C_2)=1.65\,P(C_2)+P(C_2)-0.33\,P(C_2)$$

$$1=2.23\,P(C_2)\ \Rightarrow\ P(C_2)=0.4310$$

$$P(C_1\cap C_2)=0.33\,P(C_2)=(0.33)(0.431)=0.1422$$

b) $$P(C_1\cap C_2')=P(C_1)-P(C_1\cap C_2)=(1.65)(0.4310)-0.1422=0.5690$$

13) H_L - lower side Head ; H_U - upper side Head (similarly for Tail)

DH - double headed coin

N - normal coin

a) $$P(H_L)=\frac{1}{5}\left(1+1+0+\frac{1}{2}+\frac{1}{2}\right)=\frac{3}{5}$$

b) $$P(H_L|H_U)=\frac{P(H_L\cap H_U)}{P(H_U)}=\frac{2/5}{3/5}=\frac{2}{3}$$

c) $$P(H_L^2|H_U^1)=\frac{P(H_L^2\cap H_U^1)}{P(H_U^1)}=\frac{P(H_L^2\cap H_U^1\cap DH)+P(H_L^2\cap H_U^1\cap N)}{P(H_U^1)}$$

$$=\frac{P(H_L^2|H_U^1\cap DH)P(H_U^1|DH)P(DH)+P(H_L^2|H_U^1\cap N)P(H_U^1|N)P(N)}{P(H_U^1)}$$

$$=\frac{1\cdot 1\cdot 2/5+1/2\cdot 1/2\cdot 2/5}{3/5}=\frac{5}{6}$$

d) $$P(H_L^2|H_U^2\cap H_U^1)=\frac{P(H_L^2\cap H_U^2\cap H_U^1)}{P(H_U^2\cap H_U^1)}=\frac{P(DH)}{P(H_U^2\cap H_U^1\cap DH)+P(H_U^2\cap H_U^1\cap N)}$$

$$=\frac{P(DH)}{P(H_U^2|H_U^1\cap DH)P(H_U^1|DH)P(DH)+P(H_U^2|H_U^1\cap N)P(H_U^1|N)P(N)}$$

$$=\frac{2/5}{1\cdot 1\cdot 2/5+1/2\cdot 1/2\cdot 2/5}=\frac{4}{5}$$

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e) $P(H_U^3) = P(H_U^3 \text{ after discard} \mid H_U^2 \cap H_U^1 \text{ is from DH}) \cdot P(H_U^2 \cap H_U^1 \text{ is from DH})$
$+ P(H_U^3 \text{ after discard} \mid H_U^2 \cap H_U^1 \text{ is from N}) P(H_U^2 + H_U^1 \text{ is from N})$

From the calculation in d):
$P(H_U^2 \cap H_U^1 \text{ is from DH}) = \frac{4}{5}$, $P(H_U^2 \cap H_U^1 \text{ is from N}) = \frac{1}{5}$

Case 1: We discarded a DH coin.
$P(DH) = P(DT) = \frac{1}{4}$, $P(N) = \frac{1}{2}$. $P(H_U^3 \text{ after discard}) = 1 \cdot \frac{1}{4} + 0 \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2}$
$= 1/2$

Case 2: We discarded an N coin:
$P(DH) = 1/2$, $P(DT) = 1/4$, $P(N) = 1/4$
$P(H_U^3 \text{ after discard}) = \frac{1}{2} \cdot 1 + \frac{4}{4} \cdot 0 + \frac{1}{2} \cdot \frac{1}{4} = \frac{5}{8}$

Overall:
$P(H_U^3) = \frac{1}{2} \cdot \frac{4}{5} + \frac{5}{8} \cdot \frac{1}{5} = \frac{21}{40}$

14) $U \to$ uses steroids, $+ \to$ positive test; $- \to$ negative test
$P(U) = 0.08$, $P(+|U) = 0.97$, $P(+|U') = 0.10$

a) $P(U|+) = \dfrac{P(+|U)P(U)}{P(+|U)P(U) + P(+|U')P(U')} = \dfrac{(0.97)(0.08)}{(0.97)(0.08) + (0.10)(0.92)} = 0.4575$

For two people: $(P(U|+))^2 = 0.4575^2 = 0.2903$

b) $P(U|-) = \dfrac{P(-|U)P(U)}{P(-)} = \dfrac{(0.03)(0.08)}{1 - 0.1696} = 0.0029$

For two people: $(P(U|-))^2 = 0.0029^2 = 8.353 \times 10^{-6}$

15) a) $P(B_{FM} \cap Q_{FM}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

b) $P((Q_{FM} \cap M_{FM}) \cup (Q_{QW} \cap M_{QW})) = \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{4}$

c) $P(B_{FM} \cap Q_{FM} \cap M_{FM}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{16}$

d) $P((B_{FH} \cap Q_{FM} \cap M_{QW}) \cup (B_{FH} \cap Q_{QW} \cap M_{FM}) \cup (B_{FM} \cap Q_{QW} \cap M_{FH})$

$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

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e) P (At least two meet) = 1 - P (All go to different places) =

$= 1 - \frac{1}{4} = \frac{3}{4}$

16) $p(H) = 0.4, \ p(H') = 0.6, \ p(P_i \mid H) = 0.5, \ p(P_i' \mid H) = 0.5, \ p(P_i \mid H') = 0$

$$P(H \mid P_1' \cap P_2') = \frac{P(P_1' \cap P_2' \mid H) P(H)}{P(P_1' \cap P_2' \mid H) P(H) + P(P_1' \cap P_2' \mid H') P(H')} =$$

$$= \frac{P(P_1' \mid H) P(P_2' \mid H) P(H)}{P(P_1' \mid H) P(P_2' \mid H) P(H) + P(P_1' \mid H') P(P_2' \mid H') P(H')} =$$

$$= \frac{(0.5)^2 (0.4)}{(0.5)^2 (0.4) + 1^2 (0.6)} = \frac{1}{7} = 0.1429$$