# PROBABILITY AND STATISTICS, CLASS EXERCISE 2

(1) a) Prove that

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

whenever $P(A)P(B) \neq 0$.
b) Show that, if $P(A|B) > P(A)$, then $P(B|A) > P(B)$.

(2) A circuit system is shown on the diagram below. The numbers in the boxes represent the probability that the corresponding component functions properly. Assume that the components fail independently.
i) What is the probability that the entire system works?
ii) Given that the system works, what is the probability that the component $A$ is not working?
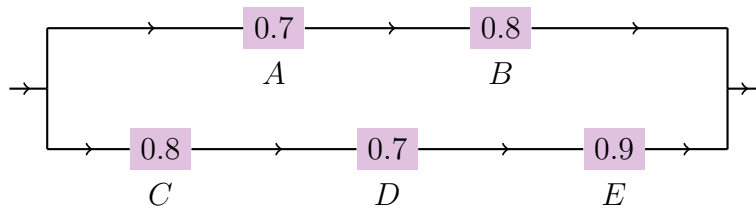


(3) The following table summarizes 204 endotermic reactions involving sodium bicarbonate.

| Final temperature | Low heat absorbtion | High heat absorbtion |
|---|---|---|
| 266 K | 12 | 40 |
| 271 K | 44 | 16 |
| 274 K | 56 | 36 |

Let $A$ denote the event that a reaction final temperatue is 271 K or less. Let $B$ be the event that the heat absorbed is high.
a) Determine the conditional probability $P(B|A)$.
b) Are the events $A$ and $B$ independent? Justify your answer numerically.

(4) What do you think of the following 'proof' by Lewis Carroll that an urn cannot contain two balls of the same color? Suppose that an urn contains two balls, each one of which is either black or white; thus $P(BB) = P(BW) = P(WB) = P(WW) = 1/4$. We add a black ball, so that $P(BBB) = P(BBW) = P(BWB) = P(BWW) = 1/4$. Next we pick a ball at random; the chance that the ball is black is (using conditional probabilities)

$$1 \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{4} = \frac{2}{3}.$$

However, if there is probability $2/3$ that a ball chosen at random from three, is black, then there must be two black and one white, which is to say that originally there was one black and one white ball in the urn.

(5) A town has two fire engines operating independently. The probability that a specific fire engine is available when needed is 0.96.
a) What is the probability that neither is available when needed?
b) What is the probability that a fire engine is available when needed?

(6) Let $A, B, C$ be independent events. Prove that the events $A \cup B$ and $C$ are independent.

(7) 42% of the citizens of Brossard own dogs and 55% own cats. The probability that a randomly selected dog owner in Brossard also owns a cat is 0.35.
a) What is the probability that a randomly selected cat owner in Brossard also owns a dog?
b) What is the probability that a randomly selected citizen of Brossard owns neither a dog nor a cat?

(8) Customers who purchase a certain make of car can order an engine in any of three sizes. Of all cars sold, 45% have the smallest engine, 35% have the medium sized one, and 20% have the largest. Of cars with the smallest engine, 10% fail an emissions test within the waranty period, while 12% of those with medium size and 15% of those with largest engine fail. A friend of yours tells you that her car, which is of this make, failed an emissions test within the waranty period. Compute the probabilities that your friend's car has small, medium or large engine.

(9) To avoid creeps from taking covert photos, iPhones bought in Japan (and seemingly also South Korea) cannot be muted when using the camera[1]. The mandatory shutter sound was installed to reduce voyeuristic "up-skirt" photography - especially in crowded places like rush-hour trains.

   The chips responsible for the camera's sound are produced at two manufacturing plants; one in Korea, and the other in Japan. The Korean factory makes 40% of the chips, while the Japanese plant produces the remaining 60%. Quality control has determined that 2% of the chips manufactured in Korea are defective, and 3% of the chips made in Japan are defective.
   (a) If a randomly selected chip is found to be defective, what is the probability that it was manufactured in Korea?
   (b) If a randomly selected chip is not defective, what is the probability that it was made in Japan?
   (c) What is the probability that a randomly selected was made in Japan or not defective?

(10) The departmental photocopier has three parts, $A, B$, and $C$ which can go wrong. The probability that $A$ will fail during a copying session is $10^{-5}$. The probability that $B$ will fail is $10^{-1}$ if $A$ fails and $10^{-5}$ otherwise. The probability that $C$ will fail is $10^{-1}$ if $A$ and $B$ both fail, $10^{-2}$ if precisely one of $A$ and $B$ fails and $10^{-5}$ otherwise. The 'Call Engineer' sign lights up if two or three parts fail. If only two parts have failed, I can repair the machine myself. But if all three parts have failed, then my attempts will only make matters worse. If the 'Call Engineer' sign light is up, and I am willing to run a risk of no more than 1% or making matters worse, should I try to repair the machine? Why or why not? Justify your argument with appropriate calculations.

(11) "Killing Season" is a British medical term used to describe the period in August when newly qualified doctors enter the National Health Service. During this time there is an up tick in medical misdiagnosis and surgical complications. A similar phenomenon exists in the US and occurs in the month of July[2].

   The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient files a lawsuit is 0.4. At any given time 15% of all doctors are facing lawsuits.

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[1]https://www.engadget.com/2016/09/30/japans-noisy-iphone-problem/
[2]https://en.wikipedia.org/wiki/July_effect

    (a) What is the probability that a doctor facing a lawsuit made a correct diagnosis.

    (b) What is the probability that a doctor facing a lawsuit made an incorrect diagnosis?

(12) The equivalents of the English saying "That's Greek to me" are: "This appears to be Spanish" (German), "This is Russian to me" (Dutch), 'It's German to me" (Philippines), "It's Hebrew" (Finnish), "It's Chinese to me" (Hebrew), "Sounds like Mars language/These are chicken intestines" (China)[3].

    A group of girls at a school are taking Advanced Cantonese which come in two modules: C1 and C2. Each girl takes only module C1, or only module C2, or both C1 and C2. The probability that a girl is taking C2 given that she is taking C1 is 0.2. The probability that a girl is taking C1 given that she is taking C2 is 0.33

    Find the probability that a girl selected at random

    (a) is taking both C1 and C2.

    (b) is taking only C1.

(13) A man has five coins: two of them are double headed, one is double tailed, and the other two are normal. He closes his eyes, picks a coin at random and tosses it.

    (a) What is the probability that the lower facing side of the coin is a head?

    (b) He opens his eyes and sees that the coin is showing heads; what is the probability that the lower face is heads?

    (c) He closes his eyes again and flips the same coin again. What is the probability that the lower face is a head?

    (d) He opens his eyes and sees that the coin is showing heads; what is the probability that the lower face is heads?

    (e) He now discards this coin, picks another at random, and flips it. What is the probability that it shows heads?

(14) In an attempt to cheat a drug test, a woman in Ohio borrowed someone else's urine and tried to pass off as her own. Unfortunately, the urine tested positive for other drugs[4]. A manufacturer claims that a drug test will detect steroid use 97% of the time. Further, 10% of all steroid-free individuals also test positive. Suppose that 8% of the rugby team members use steroids.

    a) Two of your friends on the rugby team just tested positive. What is the

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[3]https://en.wikipedia.org/wiki/Greek_to_me
[4]https://nypost.com/2018/04/24/woman-uses-someone-elses-urine-for-drug-test-fails-anyway/

probability that both of them use steroids?
b) Two of your friends on the rugby team just tested negative. What is the probability that both of them use steroids?

(15) James Bond, Q, and M have agreed to meet at a pub after work for drinks. Bond cannot remember if they agreed to meet at the "Fanny on the Hill" or at "My Father's Moustache" - so he tosses a coin to decide which pub to go to. Q is also in the same predicament; he tosses a coin to decide between "My Father's Moustache" and "The Quiet Woman". M faced with same quandary flips a coin first to decide whether or not he needs to head to the "Fanny on the Hill". If the answer is "no", then he flips again to decide between "My Father's Moustache" and "The Quiet Woman" [5] . What is the probability that

   (a) Bond and Q meet?

   (b) Q and M meet?

   (c) all three meet?

   (d) all three go to different places?

   (e) at least two meet?

(16) (Bonus) In the Middle Ages 40% of European Queens carried the gene of hemophilia. If a queen is a carrier, then each prince has a 50% chance of having hemophilia independently. If the queen is not a carrier, the prince will not have the disease. Queen B. has had two princes without the disease. What is the probability that Queen B. is a carrier?

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[5]These are real pubs in Britain. "Fanny on the Hill" is in Welling, Dartford, "My Father's Moustache" is in Louth, Lincolnshire and "The Quiet Woman" is in Earl Sterndale, Buxton