Probstat - Clex 3 - Solutions

1) a) ${}_{24}P_5 = 5100480$ b) ${}_{24}C_5 = 42504$

c) $p = \dfrac{{}_{12}P_5}{{}_{24}P_5} = 0.0186$ d) $p = \dfrac{{}_{12}C_5}{{}_{24}C_5} = 0.0186$

2) D_i - the die showing the number i; X - number of green marbles drawn

$$P(X=0) = P(X=0|D_1)P(D_1) + P(X=0|D_2)P(D_2) + P(X=0|D_3)P(D_3) + P(X=0|D_4)P(D_4)$$
$$+ P(X=0|D_5)P(D_5) + P(X=0|D_6)P(D_6) =$$
$$= \frac{{}_5C_0\,{}_5C_1}{{}_{10}C_1}\cdot\frac{1}{6} + \frac{{}_5C_0\,{}_5C_2}{{}_{10}C_2}\cdot\frac{1}{6} + \frac{{}_5C_0\,{}_5C_3}{{}_{10}C_3}\cdot\frac{1}{6} + \frac{{}_5C_0\,{}_5C_4}{{}_{10}C_4}\cdot\frac{1}{6} + \frac{{}_5C_0\,{}_5C_5}{{}_{10}C_5}\cdot\frac{1}{6} + 0\cdot\frac{1}{6} = \frac{30}{216}$$

similarly:

$$P(X=1) = \frac{1}{6}\left\{\frac{{}_5C_1\cdot{}_5C_0}{{}_{10}C_1} + \frac{{}_5C_1\cdot{}_5C_1}{{}_{10}C_2} + \frac{{}_5C_1\cdot{}_5C_2}{{}_{10}C_3} + \frac{{}_5C_1\cdot{}_5C_3}{{}_{10}C_4} + \frac{{}_5C_1\cdot{}_5C_4}{{}_{10}C_5} + \frac{{}_5C_1\cdot{}_5C_5}{{}_{10}C_6}\right\} = \frac{66}{216}$$

$$P(X=2) = \frac{1}{6}\left\{0 + \frac{{}_5C_2\cdot{}_5C_0}{{}_{10}C_2} + \frac{{}_5C_2\cdot{}_5C_1}{{}_{10}C_3} + \frac{{}_5C_2\cdot{}_5C_2}{{}_{10}C_4} + \frac{{}_5C_2\cdot{}_5C_3}{{}_{10}C_5} + \frac{{}_5C_2\cdot{}_5C_4}{{}_{10}C_6}\right\} = \frac{63}{216}$$

$$P(X=3) = \frac{1}{6}\left\{0 + 0 + \frac{{}_5C_3\cdot{}_5C_0}{{}_{10}C_3} + \frac{{}_5C_3\cdot{}_5C_1}{{}_{10}C_4} + \frac{{}_5C_3\cdot{}_5C_2}{{}_{10}C_5} + \frac{{}_5C_3\cdot{}_5C_3}{{}_{10}C_6}\right\} = \frac{43}{216}$$

$$P(X=4) = \frac{1}{6}\left\{0 + 0 + 0 + \frac{{}_5C_4\cdot{}_5C_0}{{}_{10}C_4} + \frac{{}_5C_4\cdot{}_5C_1}{{}_{10}C_5} + \frac{{}_5C_4\cdot{}_5C_2}{{}_{10}C_6}\right\} = \frac{13}{216}$$

$$P(X=5) = \frac{1}{6}\left\{0 + 0 + 0 + 0 + \frac{{}_5C_5\cdot{}_5C_0}{{}_{10}C_5} + \frac{{}_5C_5\cdot{}_5C_1}{{}_{10}C_6}\right\} = \frac{1}{216}$$

X	0	1	2	3	4	5
$p(x)$	$\frac{30}{216}$	$\frac{66}{216}$	$\frac{63}{216}$	$\frac{43}{216}$	$\frac{13}{216}$	$\frac{1}{216}$
	= 0.139	0.306	0.292	0.199	0.060	0.005

3) $|\Omega| = {}_{20}C_8$ - the total number of ways to choose 8 socks out of 20.

a) $p = \dfrac{1}{{}_{20}C_8}\cdot{}_{10}C_1\cdot{}_9C_6\cdot({}_2C_1)^6 = \dfrac{1792}{4199} = 0.4268$

b) $p = \dfrac{1}{{}_{20}C_8}\cdot{}_{10}C_2\cdot{}_8C_4\cdot({}_2C_1)^4 = \dfrac{1680}{4199} = 0.4001$

c) $p = \frac{1}{{}_{20}C_8} \cdot {}_{10}C_3 \cdot {}_7C_2 \cdot ({}_2C_1)^2 = \frac{336}{4199} = 0.0800$

d) Let i be the event that we get i complete pairs of socks.

$$P(i) = \frac{{}_{10}C_i \cdot {}_{10-i}C_{8-2i} \cdot ({}_2C_1)^{8-2i}}{{}_{20}C_8}$$

4) $\frac{22!}{9!\,5!\,4!\,4!} = 4.48 \times 10^{10}$

5) $\frac{{}_{13}C_2 \cdot ({}_4C_2)^2 \cdot {}_{11}C_1 \cdot {}_4C_1}{{}_{52}C_5} = \frac{123552}{2598960} = 0.0475$

6) a) $n = 2 \cdot 3! \cdot 3! = 72$

b) i) $n(S) = 10^4 \cdot 26^3 = 175760000$

$$p = \frac{{}_{10}P_4 \cdot {}_{26}P_3}{n(S)} = \frac{78624000}{175760000} = 0.4473$$

ii) $p = \frac{1}{4} \cdot (0.4473) = 0.1118$

7) a) $p = 1/4$

b) $p = \frac{(13!)^4 \cdot 4!}{52!} = 4.474 \times 10^{-28}$

c) $p = \frac{13! \cdot 39! \cdot 40}{52!} = 6.299 \times 10^{-11}$. Here 40 is the number of positions the diamonds could occupy.

8) a) $p = \frac{5!\,13! \cdot 14}{18!} = 1.634 \times 10^{-3}$

b) $p = \frac{5! \cdot 6! \cdot 9!}{18!} = 4.897 \times 10^{-6}$

The 9! because we have to count the permutations of the 7 Finn's plus the two other groups.

c) $p = \frac{5! \cdot 6! \cdot 7! \cdot 3!}{18!} = 4.081 \times 10^{-7}$

9) a) $p = \frac{{}_6C_1 \cdot {}_{18}C_3}{{}_{24}C_4} = 0.4607$

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b) $p(X>0) = 1 - p(X=0) = 1 - \dfrac{{}_6C_0 \cdot {}_{18}C_4}{{}_{24}C_4} = 0.7120$

c) $p = \dfrac{{}_4C_1 \cdot {}_6C_1 \cdot {}_{10}C_4}{{}_{24}C_4} = 0.2055$

10) C_i - The die shows i and i cards are selected

a) $p(W) = p(W \cap C_4) + p(W \cap C_5) + p(W \cap C_6) =$

$= \dfrac{{}_4C_4}{{}_{52}C_4} \cdot \dfrac{1}{6} + \dfrac{{}_4C_4 \cdot {}_{48}C_1}{{}_{52}C_5} \cdot \dfrac{1}{6} + \dfrac{{}_4C_4 \cdot {}_{48}C_2}{{}_{52}C_6} \cdot \dfrac{1}{6} = \dfrac{1}{77350} = 1.293 \times 10^{-5}$

b) $p(C_6 | W) = \dfrac{p(C_6 \cap W)}{p(W)} = \dfrac{1}{1.293 \times 10^{-5}} \left[\dfrac{{}_4C_4 \cdot {}_{48}C_2}{{}_{52}C_6} \cdot \dfrac{1}{6} \right] = \dfrac{5}{7}$

11) J_i - The event that i'th jar is empty

a) $p(J_1) = p(J_2) = p(J_3) = \left(\frac{2}{3}\right)^{16}$

$p(J_1 \cap J_2) = p(J_1 \cap J_3) = p(J_2 \cap J_3) = \left(\frac{1}{3}\right)^{16}$

$p(J_1 \cap J_2 \cap J_3) = 0$

$p(J_1 \cup J_2 \cup J_3)' = 1 - p(J_1 \cup J_2 \cup J_3) = 1 - \left[3\left(\frac{2}{3}\right)^{16} - 3\left(\frac{1}{3}\right)^{16} + 0\right] = 0.9954$

b) We can ignore the green and the blue marbles.

$p = {}_9C_3 \cdot {}_6C_3 \cdot {}_3C_3 \cdot \left(\frac{1}{3}\right)^9 = 0.0854$

12) X - highest ranking by a woman

$p(X=1) = \dfrac{{}_5P_1 \cdot 9!}{10!} = \dfrac{1}{2}$; $p(X=2) = \dfrac{{}_5P_1 \cdot {}_5P_1 \cdot 8!}{10!} = \dfrac{5}{18}$; $p(X=3) = \dfrac{{}_5P_2 \cdot {}_5P_1 \cdot 7!}{10!} = \dfrac{5}{36}$

$p(X=4) = \dfrac{{}_5P_3 \cdot {}_5P_1 \cdot 6!}{10!} = \dfrac{5}{84}$; $p(X=5) = \dfrac{{}_5P_4 \cdot {}_5P_1 \cdot 5!}{10!} = \dfrac{5}{252}$; $p(X=6) = \dfrac{{}_5P_5 \cdot {}_5P_1 \cdot 4!}{10!} = \dfrac{1}{252}$

X	1	2	3	4	5	6
$p(x)$	1/2	5/18	5/36	5/84	5/252	1/252