PROBABILITY AND STATISTICS, CLASS EXERCISE 3

(1) Hourly samples of a chemical process are collected for 24 samples per day, but due to the cost considerations only 5 samples are actually analysed each day. Asume the decision on which 5 samlpes are to be analysed is made at random.
a) In how many ways can the 5 analysed samples be distributed through the day if the the 5 analyses done are all different?
b) In how many ways can the 5 analysed samples be distributed through the day if the 5 analyses done are identical?
c) What is the probability that all 5 analysed samples would be from the samples collected before noon if the analyses done are all different?
d) What is the probability that all 5 analysed samples would be from the samples collected before noon if the analyses done are identical?

(2) A jar contains five green marbles and five red marbles. You roll a fair die once. Next you randomly draw (without replacement) as many marbles from the jar as shown on the number of shown on the die. Let X be the number of green marbles drawn from the jar. What is the probability mass function of X? Organize your solution into a table.

(3) Suppose that you have 10 pairs of socks in the closet. Pick 8 socks at random.
   (a) What is the probability that you get exactly one complete pair of socks?
   (b) What is the probability that you get two complete pairs of socks?
   (c) What is the probability that you get three complete pairs of socks?
   (d) Generate a formula for computing the probability of getting i complete pairs of socks.

(4) Jackie manages 22 programmers at CIBC. In how many ways could the programmers be assigned into four group projects requiring 9,5,4 and 4 programmers respectively. Each programmer is assigned to precisely one team.

(5) Two Pairs is a poker hand in which two of the cards are of the same rank, another two cards are also of the same rank, which however is different from the rank of the first pair and the fifth card is of a rank different from the ranks of the two pairs. Here is an example: $5\diamondsuit, 5\spadesuit, Q\clubsuit, Q\spadesuit, 10\diamondsuit$. What is

the probability that a randomly selected poker hand is Two Pairs?

(6) In 2006, thieves planning to steal a koala from a zoo in Australia, had to change their minds after it proved too vicious to be abducted. The perps, later confessed that their original plan was to swap the koala for drugs; but after it scratched hell out of them, they opted to steal a 1.4 metre crocodile instead - which they claimed was a lot easier[1].
   (a) A witness reported that a car seen speeding away from the zoo had a number plate that began with a V or W, the digits 4,7, and 8, and the end letters A,C, and E. However, they could not remember the order of the digits or the end letters. How many cars would need to be checked to be sure of including the suspect car?

   (b) In other parts of Australia, a licence plate consists of a sequence of seven symbols: number, letter, letter, letter, number, number, number, where a letter is any one of 26 letters (A - Z) and a number is one of $(0 - 9)$. Assume that all licence plates are equally likely.

      (i) What is the probability that all symbols are different?
      (ii) What is the probability that all symbols are different and the first number is the largest among the numbers?

(7) Shuffle a deck of 52 cards. What is the probability that
   (a) the top card is a heart?
   (b) all cards of the same suit end up next to each other?
   (c) the diamonds are together?

(8) A group of 18 Scandinavians consists of 5 Norwegians, 6 Swedes, and 7 Finns. They are seated at random in a row of chairs. Compute the following probabilities:
   (a) that all the Norwegians sit together,
   (b) that all the Norwegians and all the Swedes sit together
   (c) that all the Norwegians, all the Swedes, and all the Finns sit together.

(9) WD-40 is a low-viscosity oil used for lubricating mechanical components, removing adhesives, and dissolving dirt. The formula for WD-40 is not patented as patenting it would require the company to disclose the secret recipe which

[1]https://www.smh.com.au/national/cranky-koala-meaner-than-stolen-croc-20060330-gdn9jt.html

is used to make the solvent[2]. "WD-40" stands for "water displacement, 40th attempt".

At a chemical plant, 24 holding tanks are used to store the final product. Four tanks are selected at random and without replacement. Suppose that six of the tanks contain material in which the viscosity exceeds customer requirements.

(a) What is the probability that exactly one tank in the sample contains high-viscosity material?
(b) What is the probability that at least one tank in the sample contains high-viscosity material?
(c) In addition to the six tanks with high viscosity levels, four different tanks contain material with high impurities. What is the probability that exactly one tank in the sample contains high-viscosity material, and exactly one tank in the sample contains material with high impurities?

(10) Consider the following game. A player rolls a fair die. If he rolls 3 or less, he loses immediately. Otherwise he selects, at random, as many cards from a full deck as the number that came up on the die. The player wins if all four Aces are among the selected cards.
(a) Compute the winning probability for this game.

(b) Smith tells you that he recently played this game once and won. What is the probability that he rolled a 6 on the die?

(11) You have 16 marbles: 3 blue, 4 green, and 9 red. You also have 3 jars. For each of the 16 marbles, you select a jar at random and place the marble into it. Assume that the jars are large enough to accommodate any number of marbles in them.

(a) What is the probability that no jar is empty?

(b) What is the probability that each jar contains 3 red marbles in it?

(12) Five men and five women are ranked according to their scores on an exam. Assume that no two scores are the same and all possible rankings are equally likely. Let the random variable X be the highest ranking achieved by a women. What is the probability mass function of X? Organize your solution into a table.

[2]https://www.wd40company.com/our-company/our-history/