Probstat - Clex 4 - Solutions

(1)

x	p(x)
200000 - 1	$1/{}_{35}C_5 = 1/324632 \approx 3.08 \times 10^{-6}$
200 - 1	${}_5C_4 \cdot {}_{30}C_1 / {}_{35}C_5 = 150/324632 \approx 4.62 \times 10^{-4}$
2 - 1	${}_5C_3 \cdot {}_{30}C_2 / {}_{35}C_3 = 4350/324632 \approx 0.0134$
-1	$1 - p(\text{winning smthng}) = 1 - \frac{4501}{324632} = \frac{320131}{324632} \approx 0.9861$

$E(X) = \sum x p(x) = -0.2647$ \$

$$Var(X) = (-0.2647 - 199999)^2 \cdot \frac{1}{324632} + (-0.2647 - 199)^2 \cdot \frac{150}{324632} + (-0.2647-1)^2 \frac{4350}{324632} + (-0.2647-(-1))^2 \cdot \frac{320131}{324632} = 123234.44$$

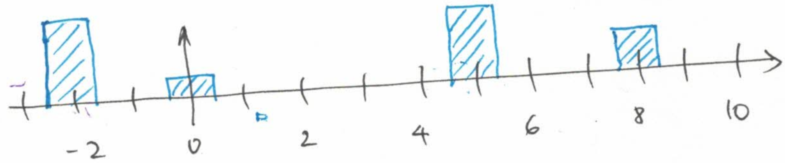
$\sigma(X) = 351.05$ \$

(2) $E(a+X) = E(a) + E(X) = a + E(X)$

$$Var(a+X) = \sum_x (a+x-E(a+X))^2 p(x) = \sum_x (\cancel{a}+x-\cancel{a}-E(X))^2 p(x) = \sum_x (x-E(X))^2 p(x) = Var(X).$$

(3)

X	-2	0	5	8
p(x)	0.41	0.08	0.33	0.18



(4) a)

X	1	2	3	4
p(x)	0.2	0.4	0.3	0.1

$E(X) = \sum x p(x) = 2.3$

$Var(X) = \sum (x-\mu) p(x) = 0.81$, $\sigma(X) = 0.9$

b)

X	45	40	35	30
p(x)	0.2	0.4	0.3	0.1

$E(X) = \sum x p(x) = 38.5$

$Var(X) = \sum (x-\mu)^2 p(x) = 20.25$ $\sigma(X) = 4.5$

(5) The number X of heads on the second round is the same as if we toss all the coins twice and count the number which shows heads on both occasions. Each coin shows heads twice with probability p^2

$$P(X = k) = {}_nC_k \, p^{2k} (1-p^2)^{n-k}$$

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(6) Binomial : $n = 45$, $p = 1/18$

a) $E(X) = np = 45 \cdot \frac{1}{12} = 3.75$

$\sigma(X) = \sqrt{np(1-p)} = \sqrt{(3.75)\frac{11}{12}} = 1.854$

b) $p(X<3) = p(0) + p(1) + p(2) =$

$= {}_{45}C_0 \left(\frac{1}{12}\right)^0 \left(\frac{11}{12}\right)^{45} + {}_{45}C_1 \left(\frac{1}{12}\right)^1 \left(\frac{11}{12}\right)^{44} + {}_{45}C_2 \left(\frac{1}{12}\right)^2 \left(\frac{11}{12}\right)^{43} = 0.2645$

(7) a) $E(X) = 2 \cdot \frac{1}{2} + 4 \cdot \frac{1}{2^2} + \dots + 2^N \frac{1}{2^N} = 1 + \dots + 1 = N$

$Var(X) = \sum x^2 p(x) - \left(\sum x p(x)\right)^2 = 2^2 \cdot \frac{1}{2} + 4^2 \cdot \frac{1}{2^2} + 8^2 \cdot \frac{1}{2^3} + \dots + (2^N)^2 \frac{1}{2^N} - N^2$

$= 2 + 4 + 8 + \dots + 2^N - N^2 = \frac{2^{N+1} - 2}{2 - 1} - N^2 = 2^{N+1} - 2 - N^2$

$\sigma(X) = \sqrt{2^{N+1} - 2 - N^2}$

N	1	2	3	4	5	6	7	8	9	10
E(N)	1	2	3	4	5	6	7	8	9	10
σ(N)	1	1.41	2.23	3.74	6.08	9.49	14.32	21.12	30.68	44.11

The standard deviation grows exponentially with N. I will not play this game.

b) $E(X) = \sum_{k=1}^{\infty} 2^k \cdot \frac{1}{2^k} = \sum_{k=1}^{\infty} 1 = \infty$

$\sigma(X) = \infty$

I will not play this game.