

PROBABILITY AND STATISTICS, CLASS EXERCISE 4

- (1) A state has a 5/35 lottery game. The ticket costs 1\$. Matching all 5 numbers (jackpot) pays 200,000\$, matching 4 of the 5 numbers pays 200\$ and matching 3 of the 5 numbers pays 2\$. The rest of the tickets are non-winning. What is the expected value (in \$) of this game and what is the standard deviation?
- (2) Show that $Var(a + X) = Var(X)$ for any discrete random variable X and constant a .
- (3) A random variable X has cumulative probability distribution given by

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.41 & -2 \leq x < 0 \\ 0.49 & 0 \leq x < 5 \\ 0.82 & 5 \leq x < 8 \\ 1 & 8 \leq x. \end{cases}$$

Determine the pmf of X and draw a histogram.

- (4) A chemical supply company currently has in stock 50kg of a certain chemical, which it sells to customers in 5kg containers. Let X be the number of containers ordered by a randomly chosen customer. Suppose that X has pmf

x	1	2	3	4
$p(x)$	0.2	0.4	0.3	0.1

- a) Compute $E(X)$, $Var(X)$ and $\sigma(X)$.
- b) Compute the expected number of kg's of chemical left after the next customer's order is shipped and the standard deviation of the number of kg's left.
- (5) We toss n coins and each one shows heads with probability p , independently of each others. Each coin which shows heads is tossed again. What is the probability mass function of the number of heads resulting from the second round of tosses?
- (6) Highway Patrol stops one of every twelve drivers who drive over the speed limit in the Airport Tunnel on HW13. In the early afternoon 750 drivers go through the Tunnel in a quarter hour and 45 of those drivers are over the speed limit.

- a) What is the expected number of drivers stopped in a quarter hour and what is the standard deviation?
- b) What is the probability that less than 3 drivers will be stopped in a quarter hour?

(7) *St. Petersburg Paradox*

You are offered to play a game of chance, where a fair coin is tossed at each stage. If tails appears on the first toss you win 2\$. If heads appears on the first toss, the coin is tossed again, but now you win 4\$ if tails appears on the second toss. In general, the game is played until tail appears for the first time and if this happens on the n 'th toss you win 2^n \$.

a) Assume that the coin could be tossed at most N times. What are your expected winnings in this game? If you have to pay an entrance fee to enter the game, what amount would you be willing to pay in order to play?

b) Assume that there is no bound on the number of tosses. What are your expected winnings in this game? Would you be willing to play this game if you have to pay a finite entrance fee?