Probstat - Clex 5 - Solutions

1) a) Negative binomial with $p=0.5$, $r=2$ successes in $x+2$ trials.

$$p(x) = {}_{x+1}C_1 \, (1-p)^{(x+2)-2} \, p^2 = {}_{x+1}C_1 \, (0.5)^x (0.5)^2 = (x+1)(0.5)^{x+2}$$

b) 4 children $\to$ 2 males, $p(2) = 3(0.5)^4 = 0.1875$

c) $E(X) = \frac{r}{p} = \frac{2}{0.5} = 4$, $\sigma(X) = \sqrt{\frac{r(1-p)}{p^2}} = \sqrt{\frac{2(0.5)}{(0.5)^2}} = 2$

2) a) Binomial, $p=0.8$, $n=15$, $x=10$

$$p(10) = {}_{15}C_{10} \, (0.8)^{10} (0.2)^5 = 0.1032$$

b) Negative binomial

$$p(15) = {}_{14}C_9 \, (0.8)^{10} (0.2)^5 = 0.0688$$

3) Hypergeometric, $N=160$, $r=75$, $n=30$

a) $E(X) = \frac{nr}{N} = \frac{30(75)}{160} = 14.06$, $\sigma(X) = \sqrt{\frac{nr(N-r)}{N^2}} \cdot \sqrt{\frac{N-n}{N-1}} = 2.4715$

b) $p(15) = \frac{{}_{75}C_{15} \cdot {}_{85}C_{15}}{{}_{160}C_{30}} = 0.1492$

c) Binomial approximation : $n=30$, $p = \frac{75}{160}$

$$p(15) \approx {}_{30}C_{15} \left(\frac{75}{160}\right)^{15} \left(\frac{85}{160}\right)^{15} = 0.1362$$

Poisson approximation : $\lambda = 30 \cdot \frac{75}{160} = 14.0625$

$$p(15) \approx \frac{e^{-14.0625} (14.0625)^{15}}{15!} = 0.0994$$

4) Poisson

a) $\lambda = 5$, $p(6) = \frac{e^{-5} \cdot 5^6}{6!} = 0.1462$

b) $\lambda = 5$, $p(X>2) = 1 - p(0) - p(1) - p(2) = 1 - \left[e^{-5} + e^{-5} \cdot 5 + e^{-5} \cdot \frac{5^2}{2!}\right] = 0.8754$

c) $\lambda = 5 \cdot 2 = 10$, $p(12) = \frac{e^{-10} (10)^{12}}{12!} = 0.0948$

d) $\lambda = 5 \cdot 3 = 15$, $p(X=15) + p(X=16) = \frac{e^{-15} \cdot 15^{15}}{15!} + \frac{e^{-15} \cdot 15^{16}}{16!} = 0.1984$

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5) a) Binomial $p = 1/80$, $n = 200$. Since p is small and n is large we can use Poisson approximation with $\lambda = np = \frac{1}{80} \cdot 200 = 2.5$

b) $P_{binom.}(X \geq 3) \approx 1 - [P_{Poisson}(0) + P_{Poisson}(1) + P_{Poisson}(2)] =$

$$= 1 - \left[e^{-2.5} + e^{-2.5}(2.5) + \frac{e^{-2.5}(2.5)^2}{2!}\right] = 0.4562$$

c) $\lambda = n/80$

$P(X \geq 1) = 1 - P(X=0) \Rightarrow 0.98 = 1 - e^{-n/80}$, $e^{-n/80} = 0.02$, $n = 313$

6) $$\alpha = \sum_{x=0}^{7} {}_{25}C_x (0.4)^x (0.6)^{25-x} = 0.1536$$

$$\beta = \sum_{x=8}^{25} {}_{25}C_x (0.3)^x (0.7)^{25-x} = 0.4881$$

7) $$\alpha = \sum_{x=25}^{40} {}_{40}C_x (0.5)^x (0.5)^{40-x} = 0.0769$$

$$\beta = \sum_{x=0}^{24} {}_{40}C_x (0.6)^x (0.4)^{40-x} = 0.5598$$

8) $$\frac{{}_kC_x \cdot {}_{N-k}C_{n-x}}{{}_NC_n} = \frac{k!}{(k-x)!\,x!} \cdot \frac{(N-k)!}{(N-k-n+x)!\,(n-x)!} \cdot \frac{(N-n)!\,n!}{N!} =$$

$$= \frac{n!}{(n-x)!\,x!} \cdot \frac{k!}{(k-x)!} \cdot \frac{(N-k)!}{(N-k-n+x)!} \cdot \frac{(N-n)!}{N!} = \{\text{Insert some } N\text{'s}\} =$$

$$= {}_nC_x \left\{\frac{k}{N} \cdot \frac{k-1}{N} \cdots \frac{k-x+1}{N}\right\} \left\{\frac{N-k}{N} \cdots \frac{N-k-n+x+1}{N}\right\} \left\{\frac{N}{N} \cdots \frac{N-n+1}{N}\right\}^{-1} =$$

$$\longrightarrow {}_nC_x \{p \cdot p \cdots p\} \{(1-p)(1-p)\cdots(1-p)\} \{1 \cdots 1\} =$$

$$= {}_nC_x\, p^x (1-p)^{n-x}$$