PROBABILITY AND STATISTICS, CLASS EXERCISE 5

(1) Suppose that the probability for female birth is $p = 0.5$. A couple wishes to have exactly two female children in their family. They will have children until this condition is fulfilled.
a) What is the probability that the family has x male children?
b) What is the probability that the family has 4 children?
c) How many children would you expect this family to have? What would be the standard deviation of the number of children?

(2) A particular traffic light on your morning commute is green 20% of the time that you approach it. Assume that each morning represents an independent trail.
a) What is the probability that the light will be red 10 times on the next 15 commutes?
b) What is the probability that the light will be red on the 15'th commute for 10th time?

(3) A batch contains 160 bacteria 75 of which are not capable of cellular replication. If you take a random sample of 30 bacteria from the batch
a) What is the expected value and the standard deviation of the number of bacteria which is not capable of replication.
b) Compute the probability that precisely 15 bacteria from the batch are not capable of replication.
c) Compute binomial and Poisson approximations for the probability that precisely 15 bacteria from the batch are not capable of replication?

(4) At a certain intersection, the average number of tickets issued for speeding is five per day.
(a) What is the probability that six tickets will be issued for speeding the next day?
(b) In one day, what is the probability that more than two tickets will be issued for speeding?
(c) What is the probability that 12 tickets for speeding will be issued over the next two days?

(d) What is the probability that 15 or 16 tickets will be issued over the next three days?

(5) Blood types are broken into four groups: A, B, AB, and O, with a positive or negative rhesus factor. Patients who are in urgent need of a blood transfusion, are often given O negative blood, because it is compatible with everyone. Since this type of blood is frequently overused and generally in short supply, people with O negative blood are seen as highly desirable blood donors.

In a large town, on average, one person in 80, has O negative blood. If 200 blood donors are sampled at random,

(a) Argue informally that the conditions to use a Poisson approximation to the Binomial are satisfied. Use the Poisson approximation to answer the questions which follow.

(b) Calculate the probability that in 200 donors, at least three people have O negative blood.

(c) How many donors must be sampled so that the probability of including at least one donor with O negative blood is 98% or more?

(6) A new car dealer tests for customers who will pay $1000 down for free financing for 4 years. A random sample of 25 buyers is taken; X is the number of customers who will take the financing deal and p is the proportion of such customers in the population. The hypothesis are as follows

$$H_0 : p = 0.4, \qquad H_1 : p < 0.4$$

a) Find the probability for type I error α if the critical region is $X < 8$.

b) Determine the probability for type II error β if the true proportion is $p_{true} = 0.3$.

(7) It is speculated that half of the ML engineers who use neural networks for image classification use pretrained models whose code the engineers do not understand. To test this hypothesis a random sample of 40 ML engineers is chosen and they are tested on their ability to explain the code for a pretrained classification network. The hypothesis are

$$H_0 : p = 0.5, \qquad H_1 : p > 0.5$$

a) Find α is the critical region is $X > 24$, where X is the number of engineers in the sample who cannot explain the code.

b) Find β if the true proportion in the population is $p_{true} = 0.6$.

(8) (Bonus) Show that the binomial distribution is a limiting case of the hypergeometric distribution: as $N \to \infty$ while $K/N \to p$ and $n/N \to 0$

$$\frac{{}_K C_x \ \cdot \ {}_{N-K} C_{n-x}}{{}_N C_n} \ \to \ {}_n C_x \ p^x (1-p)^{n-x}$$