PROBABILITY AND STATISTICS, CLASS EXERCISE 6

(1) Let X be a random variable with pdf

$$f(x) = \begin{cases} 90x^8(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Compute the cumulative distribution function of X.
b) Compute $p(0.25 < X < 0.5)$
c) Compute $E(X)$ and $Var(X)$.

(2) For what value of the parameter C is the function below a valid probability density function:

$$p(x) = Ce^{-x-e^{-x}}, x \in \mathbb{R}$$

With the correct value for C this is the pdf of the extreme-value distribution. Find the cumulative distribution function of the extreme value distribution.

(3) An ecologist wishes to mark off a circular sampling region having radius $10m$. However, the radius of the resulting region is actually a random variable R with pdf

$$f(r) = \begin{cases} \frac{3}{4}[1-(10-r)^2] & 9 < r < 11 \\ 0 & \text{otherwise} \end{cases}$$

What is the expected area of the resulting circular region?

(4) The length of time X (in hours), needed by students in HTH to complete this assignment is a random variable with PDF given by

$$f(x) = \begin{cases} k(x^2+x) & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k that makes $f(x)$ a probability density function
(b) Find the cumulative distribution function.
(c) There are 38 students in our HTH class. What is the probability that eight or nine or them will complete this assignment in under half an hour. Assume that completion times are independent.

(5) Suppose that the time it takes for a hotline operator to fill out an electronic form registering tips from concerned citizens, is uniformly between 1.5 and 11.5 minutes.
   (a) What are the mean and variance of the time it takes an operator to fill out the form?
   (b) What is the probability that it will take less than two minutes to fill out the form?
   (c) What is the probability that it will take between 5 minutes and 10 minutes to fill out the form?
   (d) 99% of the forms take less than how many minutes to fill out?
   (e) By the end of one an hour, an operator has completed 10 electronic forms. What is the probability that at least two of them took longer than 7 minutes to fill out? Assume that completion times are independent.

(6) A physicians office receives an average of six calls per hour.
   (a) What is the probability that the office will receive no calls in the next 10 minutes?
   (b) What is the probability that there will be at least one phone call within the next half hour?
   (c) If no phones calls were received at the office in the last 20 minutes, what is the probability that one will be received in the next 8 minutes?
   (d) How many minutes would the office have to wait in order to get at least one phone call with 99% probability?

(7) The distance between major cracks in a highway follows an exponential distribution with a mean of five miles.
   (a) What is the probability that there are two or three major cracks in a 10-mile stretch of the highway?
   (b) What is the probability that the distance between two successive cracks is greater than 10-miles?
   (c) What is the probability that the first major crack occurs between 12 and 15 miles of the start of inspection?
   (d) Given that there are no cracks in the first five miles inspected, what is the probability that there are no major cracks in the next 10 miles inspected?